Neutrino mixing and CP phase correlations

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A special form of the $3 \times 3$ Majorana neutrino mass matrix first appeared in 2002 [1,2], i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix},$$

where $A, B$ are real. It was shown that $\theta_{13} \neq 0$ and yet both $\theta_{23}$ and the CP nonconserving phase $\delta_{CP}$ are maximal, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm \pi/2$. Subsequently, this pattern was shown [3] to be protected by a symmetry, i.e. $e \leftrightarrow e$ and $\mu \leftrightarrow \tau$ exchange with CP conjugation. All three predictions are consistent with present experimental data. Recently, a radiative (scotogenic) model of inverse seesaw neutrino mass has been proposed [4] which naturally obtains

$$\mathcal{M}_\nu^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix} \mathcal{M}_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix},$$

where $\lambda = f_\tau/f_\mu$ is the ratio of two real Yukawa couplings.

This model has three real singlet scalars $s_{1,2,3}$ and one Dirac fermion doublet $(E^0, E^-)$ and one Dirac fermion singlet $N$, all of which are odd under an exactly conserved (dark) $Z_2$ symmetry. As a result, the third one-loop radiative mechanism proposed in 1998 [5] for generating neutrino mass is realized, as shown in Fig. 1.

The mass matrix linking $(\bar{N}_L, \bar{E}^0_L)$ to $(N_R, E^0_R)$ is given by

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N \\ m_D \\ m_E \end{pmatrix},$$

where $m_N, m_E$ are invariant mass terms, and $m_D, m_F$ come from the Higgs vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, $N$ and $E^0$ mix to form two Dirac fermions of masses $m_{1,2}$, with mixing angles

$$m_D m_E + m_T m_N = \sin \theta_L \cos \theta_L (m_1^2 - m_2^2),$$

$$m_D m_N + m_T m_E = \sin \theta_R \cos \theta_R (m_1^2 - m_2^2).$$

To connect the loop, Majorana mass terms $(m_L/2)N_L N_L$ and $(m_R/2)N_R N_R$ are assumed. Since both $E$ and $N$ may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw [6–8] as explained in Ref. [4]. Using the Yukawa interaction $f_\nu \bar{E}^0_L \nu_L$, the one-loop Majorana neutrino mass is given by

$$m_\nu = f^2 m_R \sin^2 \theta_R \cos^2 \theta_R (m_1^2 - m_2^2)^2 \times \int \frac{d^4k}{(2\pi)^2} \frac{k^2}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_T^2)^2} + f^2 m_L m_1^2 \sin^2 \theta_L \cos^2 \theta_L \int \frac{d^4k}{(2\pi)^2} \frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)(k^2 - m_T^2)^2}$$
\[ E_\alpha = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \quad E_\beta = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3} \end{pmatrix}, \]

\[ M_d = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \]

Hence
\[ M_{\nu} M_{\nu}^\dagger = E_\alpha U M^2_d U^\dagger E_\alpha^\dagger. \]

where
\[ \Delta = U^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \lambda - 1 \end{pmatrix} U, \quad M^2_{\text{new}} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & \lambda^2 m_3^2 \end{pmatrix}. \]

We now diagonalize numerically
\[ [1 + \Delta] M^2_{\text{new}} [1 + \Delta^\dagger] = O M^2_{\text{new}} O^\dagger, \]
where \( O \) is an orthogonal matrix, and \( M^2_{\text{new}} \) is diagonal with mass eigenvalues equal to the squares of the physical neutrino masses. Let us define
\[ A = (1 + \Delta)^{-1} O, \]
then
\[ A M^2_{\text{new}} A^\dagger = M^2_d. \]

Since \( U \) is known with \( \theta_{23} = \pi/4 \) and \( \delta = \pm \pi/2 \), we know \( \Delta \) once \( \lambda \) is chosen. The orthogonal matrix \( O \) has three angles as parameters, so \( A \) has three parameters. In Eq. (14), since the three physical neutrino mass eigenvalues of \( M^2_{\text{new}} \) are given, the three off-diagonal entries of \( M_d \) are constrained to be zero, thus determining the three unknown parameters of \( O \). Once \( O \) is known, \( U O \) is the new neutrino mixing matrix, from which we can extract the correlation of \( \theta_{23} \) with \( \delta_{\text{CP}} \). There is of course an ambiguity in choosing the three physical neutrino masses, since only \( \Delta m_{32}^2 \) and \( \Delta m_{21}^2 \) are known. There are also the two different choices of \( m_1 < m_2 < m_3 \) (normal ordering) and \( m_3 < m_1 < m_2 \) (inverted ordering). We consider each case, and choose a value of either \( m_1 \) or \( m_3 \) starting from zero. We then obtain numerically the values of \( \sin^2(2\theta_{12}) \) and \( \delta_{\text{CP}} \) as functions of \( \lambda \neq 1 \). We need also to adjust the input values of \( \theta_{12} \) and \( \theta_{13} \), so that their output values for \( \lambda \neq 1 \) are the preferred experimental values.

We use the 2014 Particle Data Group values [9] of neutrino parameters:
\[ \sin^2(2\theta_{12}) = 0.846 \pm 0.021 \]
\[ \Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2 \]
\[ \sin^2(2\theta_{13}) = 0.999 \left( \begin{array}{c} +0.001 \\ -0.018 \end{array} \right) \]
\[ \Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (normal)}, \]
\[ \sin^2(2\theta_{23}) = 1.000 \left( \begin{array}{c} +0.000 \\ -0.017 \end{array} \right), \]
\[ \Delta m_{32}^2 = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \text{ (inverted)}, \]
\[ \sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}. \]

We consider first normal ordering, choosing the three representative values \( m_1 = 0.03, 0.06 \text{ eV} \). We then vary the value of \( \lambda > 1 \). [The case \( \lambda < 1 \) is equivalent to \( \lambda^{-1} > 1 \) with \( \mu-\tau \) exchange.] Following the algorithm already mentioned, we obtain numerically the values of \( \sin^2(2\theta_{23}) \) and \( \delta_{\text{CP}} \) as functions of \( \lambda \). Our solutions are fixed by the central values of \( \Delta m_{21}^2 \), \( \Delta m_{32}^2 \), \( \sin^2(2\theta_{12}) \), and \( \sin^2(2\theta_{13}) \). In Figs. 2 and 3 we plot \( \sin^2(2\theta_{23}) \) and \( \delta_{\text{CP}} \) respectively versus \( \lambda \). We see from Fig. 2 that \( \lambda < 1.15 \) is required for \( \sin^2(2\theta_{23}) > 0.98 \). We also see from Fig. 3 that \( \delta_{\text{CP}} \) is not sensitive to \( m_1 \). Note that our scheme does not distinguish \( \delta_{\text{CP}} \) from \( -\delta_{\text{CP}} \). In Fig. 4 we plot \( \sin^2(2\theta_{23}) \) versus \( \delta_{\text{CP}} \). We see that \( \delta_{\text{CP}}/(\pi/2) > 0.95 \) is required for \( \sin^2(2\theta_{23}) > 0.98 \).
We then consider inverted ordering, using $m_3$ instead of $m_1$. We plot in Figs. 5, 6, and 7 the corresponding results. Note that in our scheme, the effective neutrino mass $m_{ee}$ measured in neutrinoless double beta decay is very close to $m_1$ in normal ordering and $m_3 + \sqrt{\Delta m^2_{32}}$ in inverted ordering. We see similar constraints on $\sin^2(2\theta_{23})$ and $\delta_{CP}$. In other words, our scheme is insensitive to whether normal or inverted ordering is chosen. Finally, we have checked numerically that $\theta_{23} < \pi/4$ if $\lambda > 1$, and $\theta_{23} > \pi/4$ if $\lambda < 1$. As we already mentioned, the two solutions are related by the mapping $\lambda \to \lambda^{-1}$.

In conclusion, we have explored the possible deviation from the prediction of maximal $\theta_{23}$ and maximal $\delta_{CP}$ in a model of radiative inverse seesaw neutrino mass. We find that given the present $1\sigma$ bound of 0.98 on $\sin^2(2\theta_{23})$, $\delta_{CP}/(\pi/2)$ must be greater than about 0.95.

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References