PROBLEM OF FLAVOUR IN SUSY GUT
AND HORIZONTAL SYMMETRY

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Abstract

The concept of non-abelian horizontal symmetry $SU(3)_H$ can greatly help in understanding the fermion and sfermion flavour structures in supersymmetric grand unification. For the sake of demonstration the $SU(5) \times SU(3)_H$ model, suggested earlier in ref. [1], is revisited. We show that under very simple and natural assumption it links the sfermion mass pattern to those of fermions in a remarkable way. All dangerous supersymmetric flavour-changing contributions are naturally suppressed in a general case, independently of the concrete texture for fermion mass matrices. Nevertheless, within this framework we present an example of predictive model for fermion masses and mixing, which leads to 7 consistent predictions for the low energy observables.

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1 Introduction

The two most promising ideas beyond the Standard Model, supersymmetry (SUSY) and grand unification theory (GUT), can be united by a simple relation

\[ \text{SUSY + GUT = LOVE} \] (1)

In support of this formula one can mention the potential of SUSY to solve the personal problems of GUT (gauge hierarchy and doublet-triplet splitting), an impressive fact of the MSSM gauge couplings unification in minimal \( SU(5) \), \( b - \tau \) Yukawa unification and its impact on top mass, etc.

There are, however, family (flavour) problems, which in the context of supersymmetric theory have two aspects: fermion flavour and sfermion flavour. First aspect, questioning the origin of family replication, quark and lepton mass spectrum and mixing, etc. (for a review, see ref. \cite{2}), has no appealing solutions within the naive (minimal) LOVE models where the Yukawa constants remain arbitrary (hereafter LOVE is defined by eq. (1)). However, there are realistic and predictive LOVE frameworks based on \( SO(10) \) \cite{3} or \( SU(6) \) \cite{4} symmetries which provide satisfactory explanations to the fermion flavour features.

Second aspect, which is a specific of SUSY, questions the origin and pattern of the soft SUSY breaking (SSB) terms, or in other words the mass and mixing spectrum of superpartners. There are no direct experimental data on sfermion mass pattern. Theoretical arguments based on the the Higgs mass stability tell us that these should be of few hundred GeV, or may be few GeV. On the other hand, since masses of particles and sparticles have principally different origin, there is no physical reason that the particle-sparticle couplings to neutral gauginos should be diagonal in a basis of mass eigenstates. Thus in general there should be dramatic supersymmetric contributions to rates of the flavour changing (FC) processes like \( \mu \to e + \gamma \) decay, \( K^0 - \bar{K}^0 \) transition, etc., much exceeding the rates predicted by the standard model in agreement with experiment. Therefore, suppression of such FC transitions in experiment puts very strong constraints on the mass and mixing spectrum of yet undiscovered sparticles.

Although in the MSSM natural suppression of the SUSY FC phenomena can be achieved by assuming the universal soft SUSY breaking \cite{5}, in the context of the LOVE this idea becomes insufficient since the effects of physics beyond the GUT scale can strongly violate the soft-terms universality at low scales \cite{6, 8}. These has most dramatic impact on the realistic LOVE frameworks like \cite{3, 4}. Such theories above the GUT scale \( M_X \approx 10^{16} \text{GeV} \) contain, besides the standard chiral set of the ‘would be light’ fermions \( f = (u, d); (\nu, e); u^c, d^c, e^c \), also a vector-like set of \( F \)-fermions \( F + \bar{F} \) in representations similar to that of \( f \)’s. All these states \( f, F, \bar{F} \) interact with constants typically \( \sim 1 \) with various Higgs superfields which break GUT symmetry down to \( SU(3) \times SU(2) \times U(1) \) and also induce large mass terms between various fermion states. Actual identification of the light physical states \( f' \) of the MSSM occurs after integrating out the heavy sector \cite{9} at the GUT scale, and \( f' \) generically are some linear combinations of the original \( f \) and \( F \) states. These feature provides appealing explanation of the origin of fermion flavour, since the small Yukawa constants in MSSM can be understood as ratios of different physical scales present in the theory beyond the electroweak scale. However, the same feature creates severe problems is sfermion sector, due to presence of the large Yukawa couplings above the GUT scale which generically should not be diagonal in the basis of the physical low energy eigenstates \( f' \) and thus induce the flavour violation. The FC effects in the sparticle
sector do not decouple: even if the SSB terms Planck scale universality is assumed, due to the renormalization group (RG) running effects down from $M_{Pl}$ different sfermions will have different (and nondiagonal) soft masses at the (GUT) scale where the heavy $F$-states decouple. As a result, at lower energies the physical sfermions $\tilde{f}'$ will arrive being already strongly split between different families, and disoriented relative to fermion $f'$ basis. This would give rise to the dramatic FC effects unless the SUSY breaking scale is much above the TeV scale. The latter case, however, would cancel the advantage of supersymmetry in stabilizing the Higgs boson mass. Therefore, the SUSY flavour-changing problem poses a serious challenge for intimacy of the two $[\Pi]$. 

A natural way to approach (both) flavour problems is to consider theories with horizontal (inter-family) symmetry. The chiral $SU(3)_H$ symmetry unifying all fermions in horizontal triplets $[\Pi]$ seems to be most attractive for understanding the family replication. Here we show that it has a great potential to provide a coherent picture of the particle and sparticle masses and naturally solve the SUSY FC problem in GUTs.

(Several other possibilities based on discrete, abelian or the particle and sparticle masses and naturally solve the SUSY FC problem in GUTs. $[\Pi]$)

$\bar{\eta}$ triplets), for this purpose we revisit a model based on the $SU(5) \times SU(3)_H$ symmetry which was suggested in ref. $[\Pi]$. The model contains $f$ fermions in representations $(5 + 10, 3)$:

$$5_{\alpha} = (d^c, l)_\alpha, \quad 10_{\alpha} = (u^c, q, e^c)_{\alpha}$$

($\alpha = 1, 2, 3$ is $SU(3)_H$ index). Since the fermion mass terms transform as $3 \times 3 = 6 + 3$, they cannot have renormalizable Yukawa couplings to Higgses $H = (T, H_2)$, $\bar{H} = (\bar{T}, H_1)$ in $(5 + 5, 1)$ representations, where $H_{1,2}$ are the MSSM Higgs doublets and $\bar{T}, T$ are their colour-triplet partners. Therefore, fermion masses can be induced only via higher order operators $[\Pi]$:

$$\frac{g_n \chi^{\alpha \beta}_n}{M} 10_\alpha 10_\beta H, \quad \frac{g_n \chi^{\alpha \beta}_n}{M} 10_\alpha 5_\beta \bar{H},$$

where $M \gg M_W$ is some cutoff scale (hereafter to be referred as a flavour scale) and $\chi^{\alpha \beta}_n$ denotes a set of ‘horizontal’ Higgs superfields in two-index symmetric or antisymmetric representations of $SU(3)_H$: (anti)sextets $\chi^{(\alpha \beta)}$ and triplets $\chi^{[\alpha \beta]} \sim \bar{\varepsilon}^{\alpha \beta \gamma} \chi_{\gamma}$, with VEVs $\langle \chi_\alpha \rangle \leq M$. These operators can be effectively induced via exchanges of the heavy (with mass $\sim M$) $F$-fermions in representations $(10 + 5, 3)$ and $(\overline{10} + 5, 3)$:

$$X^\alpha = (U^c, Q, E^c)_{\alpha}, \quad \bar{X}^\alpha = (U, Q^c, E)_{\alpha}$$

$$V^\alpha = (D^c, L^c)_{\alpha}, \quad V^\alpha = (D, L)_{\alpha}$$

(following ref. $[\Pi]$, we use roman numerals $V = 5$ and $X = 10$ to denote their dimensions). Notice, that actual global chiral symmetry of terms $[3]$ is $U(3)_H = SU(3)_H \times U(1)_H$, where $U(1)_H$ is related to the phase transformations $5, 10 \to e^{i \omega 5}, 10$; $\chi_n \to e^{-2i \omega} \chi_n$. $U(1)_H$ can serve as Peccei-Quinn symmetry unless it is explicitly broken in the potential of $\chi_n$. $[\Pi]$. 

This picture suggests that observed mass hierarchy may emerge from the hierarchy of the $U(3)_H$ symmetry breaking VEVs, while the form of these VEVs can provide certain predictive textures for fermion mass matrices. For example, if horizontal Higgses $\chi_n$ are chosen as the sextet $\chi$ with a VEV towards $(3,3)$ component, and two triplets $\eta$ and $\xi$ having VEVs respectively towards $1^{st}$ and $3^{rd}$ components $[\Pi]$:

$$\langle \chi^{\alpha \beta} \rangle = C \delta^{\alpha}_{3} \delta^{\beta}_{3}, \quad \langle \eta_\gamma \rangle = B \delta^{\gamma}_{1}, \quad \langle \xi_\gamma \rangle = A \delta^{\gamma}_{3}$$

(5)}
then matrix of their VEVs in whole has a form

\[ \hat{V}_H = \sum_n \langle \chi_n \rangle = \begin{pmatrix} 0 & A & 0 \\ -A & 0 & B \\ 0 & -B & C \end{pmatrix} \]

(6)

Being projected on the fermion mass pattern via operators (3), this pattern leads to the Fritzsch texture \[14\]. (Although the Fritzsch ansatz for fermion masses is already excluded by experiment, its viable modification will be presented in sect. 3.) The inter-family mass hierarchy can emerge from the VEV hierarchy \( C \gg B \gg A \) in breaking the chiral \(U(3)_H\) symmetry

\[ U(3)_H \to U(2)_H \to U(1)_H \to I \]

(7)

rather than due to \textit{ad hoc} choice of small Yukawa constants.

Obviously, the horizontal symmetry can be a good tool also for understanding the sfermion mass pattern. In general, it would provide the inter-family \(SU(3)_H\) degeneracy of the SSB mass terms at the flavour scale \(M\). However, if the \(F\)-fermions contain all states in (4), then inter-family splitting between masses of physical sfermions \(f'\) will occur after integrating out the heavy sector (though with the relatively small magnitudes of splittings given by the fermion mass ratios between different families), and the sfermion mass matrices will be generically disoriented from the fermion mass matrices (by angles of the order of the CKM angles). However, even under such a "softening" of the problem, the FC effects will be persistent and dangerous unless the sfermion masses are of several TeV.

On the other hand, one can observe that the \(X + \bar{X}\) fermions contain all fragments which are needed for generation of all quark and lepton masses. Therefore, in principle the 5-plets \(\bar{V} + V\) are not necessary. Below we show that if the latter are not introduced (or decoupled at very high scales), then fermion and sfermion mass matrices induced after integrating out the \(X + \bar{X}\) states exhibit remarkable correlations, and the SUSY flavour-changing contributions are naturally suppressed.

### 2 Fermion and sfermion masses

Let us consider a theory which contains the \(F\)-states only in \(X + \bar{X}\) (4). The most general renormalizable Yukawa terms have a form:

\[ W = g_{10}XH + f_{5}XH + X\Sigma X + \bar{X}\chi_n 10 \]

(8)

(coupling constants \(\sim 1\) are understood also in the last two terms), where \(\chi_n\) denotes a set of horizontal Higgs superfields transforming as \((R, r)\) representations of \(SU(5) \times SU(3)_H\), with \(R \subset 10 \times \bar{10} = 1 + 24 + 75\) and \(r \subset 3 \times \bar{3} = 3 + \bar{6}\). Let us take for simplicity \(\Sigma\) as a singlet, though in principle it can be in any \((R, r)\) representation with \(R = 1, 24, 75; r = 1, 8\). Let us also assume that \(\langle \chi_n \rangle < \langle \Sigma \rangle = M \sim M_X\).

After substituting these VEVs in (3), the superpotential reduces to the field-dependent mass matrices for charged leptons and down quarks:

\[ l \begin{pmatrix} e^c & E^c \\ 0 & \tilde{f}H_1 \end{pmatrix}, \quad q \begin{pmatrix} d^c & Q^c \\ 0 & \tilde{M}_q \end{pmatrix} \]

\[ M_e^T \tilde{M}, \quad \tilde{M}_q \]

(9)
and for upper quarks:

\[
\begin{pmatrix}
u^c & U^c & Q^c \\
q & 0 & \hat{g} H_2 & \hat{M}_q \\
U & \hat{M}_u^T & \hat{M} & 0 \\
Q & \hat{g} H_2 & 0 & \hat{M}
\end{pmatrix}
\]

(10)

where each entry is \(3 \times 3\) matrix in itself. As long as \(\hat{g} = g \mathbb{1}, \hat{f} = f \mathbb{1}\) and \(\hat{M} = M \mathbb{1}\) are flavour blind (unit) matrices, all the information on the fermion mass and mixing pattern is contained in the off-diagonal blocks \(\hat{M}_{q,u,e} = \sum \zeta^n_q,u,e \langle \chi_n \rangle\), where the Clebsch factors \(\zeta^n\) depend on the \(SU(5)\) content of the fields \(\chi_n(R,r)\):

- \(R = 1: \zeta_q = \zeta_u = \zeta_e\),
- \(R = 24: \zeta_q = -\frac{1}{6} \zeta_e, \zeta_u = -\frac{2}{3} \zeta_e\),
- \(R = 75: \zeta_q = -\frac{1}{3} \zeta_e, \zeta_u = \frac{1}{3} \zeta_e\)

(11)

Here we do not impose any constraint on the horizontal VEV pattern, assuming only that matrices \(\hat{M}_{q,u,e}\) have the hierarchical structure resembling that of the fermion mass matrices. Implications of the ‘Fritzsch’ texture for the horizontal VEVs will be considered in next section.

After integrating out the heavy states theory reduces to the MSSM with the light fermion superfields \(f' (f' = q', l', u'_c, d'_c, e'_c)\) which are linear combinations of the initial \(f\) and \(F\) states. Notice, that \(d^c\) and \(l\) (members of \(\bar{\eta}\)) do not mix heavy states, due to absence of \(V + \bar{V}\) states. As for the components of 10: \(q, u^c, e^c\), they mix the corresponding \(F\)-states in \(X\) and in ‘seesaw’ limit \(\hat{M}_{q,u,e} \ll M\) the states which remain light are \(q' \simeq q - \hat{M}_q \hat{M}^{-1} Q\), etc. This yields the following form of the MSSM Yukawa constant matrices below the flavour scale \(M\) [1]:

\[
\hat{\lambda}^d = f \hat{M}_q \hat{M}^{-1} f, \quad \hat{\lambda}^e = f \hat{M}^{-1} \hat{M}_e^T f \\
\hat{\lambda}^u = g (\hat{M}_q \hat{M}^{-1} + \hat{M}^{-1} \hat{M}_u^T)
\]

(12)

Consider now the SSB terms, we do not assume that they are universal at the flavour scale \(M\). Thus, in general trilinear SSB terms have a form repeating all structures present in superpotential:

\[
[A10XH + A' 5X\bar{H} + A''X\Sigma\bar{X} + A_y \bar{X} \chi_n 10]_F
\]

(13)

but *dimensional* coefficients \(A, A'\) etc. are not proportional to the Yukawa constants in (8). After substituting the horizontal VEVs, these will reduce to terms analogous to (8) and (9), but with respectively modified entries. After integrating out the heavy \(F\)-sector, we see that trilinear SSB terms for the \(f'\) states like \(\bar{q}\hat{A}_u \bar{u}_c H_1\), etc. (hereafter we omit \(f'\) for the light (MSSM) states) are aligned to the corresponding Yukawa matrices:

\[
\hat{A}_u = \frac{A}{g} \hat{\lambda}_u, \quad \hat{A}_d = \frac{A'}{f} \hat{\lambda}_d, \quad \hat{A}_e = \frac{A'}{f} \hat{\lambda}_e
\]

(14)
The SSB mass terms of the $f$ and $F$ states at the scale $M$ are not universal moreover, however they are degenerated between families due to horizontal $SU(3)_H$ symmetry:

$$
m_5^2(\tilde{d}_c^+ \tilde{d}_c^- + \tilde{l}^+ \tilde{l}^-) + m_{10}^2(\tilde{u}_c^+ \tilde{u}_c^- + \tilde{\ell}_c^+ \tilde{\ell}_c^- + \tilde{q}^+ \tilde{q}^-) + m_X^2(\tilde{U}_c^+ \tilde{U}_c^- + \tilde{E}_c^+ \tilde{E}_c^- + \tilde{Q}^+ \tilde{Q}^-) + \ldots
$$

(15)

(if $M < M_X$, the soft masses actually will not be $SU(5)$-invariant, but this is not relevant for our discussion.) After integration out the heavy sector, soft mass terms of the physical $f'$ states will get family dependent contributions due to mixing between the $f$ and $F$ states. Taking into account eq. (12), we obtain that the sfermion mass terms have the following form:

$$
\mathcal{M}_{\tilde{e}_c}^2 = m_{10}^2 \left( \mathbb{1} + \frac{\delta}{2} \hat{\lambda}_c^+ \hat{\lambda}_c \right), \quad \mathcal{M}_{\tilde{l},dc}^2 = m_5^2 \mathbb{1}
$$

$$
\mathcal{M}_{\tilde{u}_c}^2 = m_{10}^2 \left( \mathbb{1} + \delta \hat{\lambda}_c^+ \hat{\lambda}_c \right), \quad \hat{\lambda} = \frac{\tilde{\lambda}_c^2}{g} - \frac{\tilde{\lambda}_c^2}{f}
$$

$$
\mathcal{M}_d^2 = m_{10}^2 \left( \mathbb{1} + \frac{\delta}{2} \hat{\lambda}_d^+ \hat{\lambda}_d \right)
$$

(16)

where $\delta = (m_X^2 - m_{10}^2)/m_{10}^2$ is generally $\sim 1$.

As we see, mass terms of the right down squarks $\tilde{d}_c$ and left sleptons $\tilde{l}$ remain degenerate between families at the flavour scale $M$. The reason is that the $d_c$ and $l$ fragments do not mix $F$-states at the decoupling of the heavy sector and thus their soft mass terms maintain the $SU(3)_H$ symmetry. Already this fact would be enough to suppress the dominant supersymmetric contributions to the $K^0 - \bar{K}^0$ transition (gluino-mediated boxes involving $\tilde{d}_c$ and $\tilde{d}$ states) and $\mu \to e\gamma$ decay (loops involving $\tilde{l}$ states).

However, in fact flavour conservation appears to be even more persistent. Eq. (16) shows that the left down squarks $\tilde{d}_c \subset q$ and right sleptons $\tilde{\ell}_c$ are split between families, but their mass matrices are aligned to the mass matrices $\hat{\lambda}_d$ and $\hat{\lambda}_\ell$ of the corresponding fermion states. Hence, no flavour changing presents in these sectors.

As for the upper squarks $\tilde{u}_c$ and $\tilde{u} \subset q$, they are not aligned with $\hat{\lambda}_u$. Therefore, the FC effects will emerge which contribute $D^0 - \bar{D}^0$ transition, etc. However, the pattern given by eq. (16) for upper squarks splitting and mixing is by no means in contradiction with the current experimental limits. In fact, $\mathcal{M}_q^2$ is alligned to $\hat{\lambda}_d$, so that the mixing angles in $u$-$\bar{u}$-gluino couplings will coincide with the CKM angles. The $u$-$\bar{u}$-gluino mixing is generally more complicated. Interestingly, in the concrete model suggested in sect. 3, the right up squarks $\tilde{u}_c$ appear to be degenerated between the first and second families. This will lead to further suppression of the FC effects in the up quark sector.

Thus, in the presented framework all dangerous FC effects are properly suppressed. However, in severe reality these could be induced by the following:

(i) If $M > M_X$, then owing to the large top Yukawa coupling the lepton flavour violation $\mu \to e\gamma$, $\tau \to \mu\gamma$, etc. can be induced due to colour triplet $T$ contribution to the RG running from $M$ down to $M_X$ [8]. However, this effect is relevant only for $\tilde{\ell}_c$ states of the third family and presently do not pose any problem.

(ii) some FC effects could emerge if the seesaw limit is not good in decoupling of the heavy $F$-states (see in the concrete model presented in next section). However,
the corrections to seesaw approximation are relevant only for the third family and thus should not cause severe problems.

(iii) MSSM contributions related to the radiative corrections below the flavour scale \( M \) down to the electroweak scale. Although in our case the boundary conditions for sfermion mass pattern are different from the universal boundary conditions generally adopted in MSSM, it is clear that these effects are still under control.

(iv) FC can be induced by truly non-renormalizable SUSY breaking F- and D-terms cutoff by \( M_{Pl} \), or the similar terms which could be induced after integrating out some other F-states with mass \( \Lambda > M \). However, these effects can be suppressed at the needed degree by assuming that \( M \ll M_{Pl} \) (or \( \Lambda \)).

Let us conclude this section with the following remark. In the above we assumed that some of the \( \chi \) fields are in the mixed representations of the \( SU(5) \times SU(3)_H \). For example, in the case of the fields \( \chi_{\alpha} \) having VEVs pattern \((\bar{3})\) it is natural to take the sextet \( \chi \) as \( SU(5) \) singlet: this will lead to the \( b - \tau \) Yukawa unification at the GUT scale. However, \( \eta \) and \( \xi \) should be taken in representations \((24,3)\) or \((75,3)\), in order to produce the nontrivial Clebsch factors which would distinguish the corresponding mass entries between quarks and leptons. However, it would be more economic to think that actually all horizontal scalars \( \chi, \eta, \xi \) are the \( SU(5) \) singlets, and the non-trivial Clebsch factors emerge due to the higher order operators like \( \frac{1}{4} \lambda^{10}\Phi \bar{\chi} X \), where \( \Phi \) is say standard 24-plet superfield of \( SU(5) \). Clearly, such operators can effectively emerge after integrating out some other heavy \( F \)-states in representations \( 10 + \bar{10} \) with masses \( \Lambda \gg M, M_X \). In general, this can violate the inter-family universality of the soft mass terms at the scale \( M \). But not necessarily. In particular, there are two possibilities to introduce such states and their interactions:

\[(A) \ X'_\alpha + \bar{X}^{\prime \alpha} : \ 10\Phi \bar{X} + \Lambda \bar{X}'X + X'\chi \bar{X} \]
\[(B) \ X^{\prime \alpha} + \bar{X}'_{\alpha} : \ 10\chi \bar{X}' + \Lambda \bar{X}'X + X'\Phi \bar{X} \quad \text{(17)}\]

Clearly, in the case (B) the sfermion states in 10 can get relatively large non-universal contributions, since the different families in 10 mix the states in \( X' \) with different angles. This could induce the considerable flavour violation. However, in the case (A) mixing between the states in 10 and \( X' \) is universal in families, at least in leading approximation. Therefore, the mass terms in (A) will not receive large family dependent contributions and remain degenerate. Concluding, although the group-theoretical structure of the considered effective operator does not depend on the pattern and way of exchange of \( F \)-states, the soft mass terms pattern is sensitive with respect to the latter.

3 An example of consistent and predictive model

In the framework presented above the solution of the flavour changing problem practically does not depend on the concrete structure of the fermion mass matrices (i.e. \( SU(3)_H \) breaking VEVs pattern). However, one can already notice that considered general framework is capable to produce the predictive ansatzes for fermion masses, which capability is rather resembling that of the \( SO(10) \) model. Indeed, since the mass matrices of all quark and lepton masses emerge from the same \( F \)-fermion exchange, they will actually differ only by the Clebsch structure of the ”vertical” Higgs...
VEVs responsible for the GUT symmetry breaking - as it in fact takes place in $SO(10)$ model.

Here we would like to present an example of consistent and predictive model built along the lines discussed above. The requirements to be fulfilled are:

(i) Natural doublet-triplet splitting

(ii) Predictivity in fermion mass and mixing pattern

(iii) Predictivity in sfermion mass and mixing and natural suppression of the SUSY FC contributions

(iv) Natural suppression of the proton decaying $d = 5$ operators (which in certain sense are the part of the whole flavour changing problem).

Let us start from the first requirement. In order to achieve the solution to the doublet-triplet splitting problem, we extend the vertical symmetry to $SU(5) \times SU(5)'$ [15]. The model involves the Higgs superfields $\Omega_a(\bar 5, 5, 1)$ and $\bar\Omega_a(5, \bar 5, 1)$ (a = 1, ..4), with VEVs of the following possible structures:

\[
\langle \Omega_1 \rangle = M_1 \cdot \text{diag}(1, 1, 1, 1, 1), \\
\langle \Omega_2 \rangle = M_2 \cdot \text{diag}(0, 0, 0, 1, 1), \\
\langle \Omega_3 \rangle = M_3 \cdot \text{diag}(1, 1, 1, 0, 0), \\
\langle \Omega_4 \rangle = M_4 \cdot \text{diag}(y, y, y, 1, 1), \quad y \neq 1
\]  

In order to maintain the gauge coupling unification, it is natural to assume that $M_1 \gg M_2, M_3, M_4 \sim M_X$. In this case $SU(5) \times SU(5)'$ at the scale $M_1$ first reduces to the diagonal $SU(5)$ subgroup, which then breaks down to $SU(3) \times SU(2) \times U(1)$ at the scale $M_X$. Let us also introduce Higgses in representations $H(5, 1) + \bar H(\bar 5, 1)$ and $H'(1, 5) + \bar H'(1, \bar 5)$. Imagine now that these are coupled to the $\Omega$ fields through the following terms in superpotential:

\[
\bar H \Omega_{1(4)} H' + H \bar\Omega_3 H'
\]  

after substituting the VEVs (18), we see that just two MSSM Higgs doublets remain light: $H_2 \subset H$ and $H_1 \subset H'$, while all colour triplets and other couple of doublets get masses $\sim M_X$. The doublet-triplet splitting is achieved in this way.

Let us introduce now the horizontal $SU(3)_H$ symmetry. We assume that the fermion superfields $5_\alpha$ and $10_\alpha$ [13] belong to representations $(5 + 10, 1, 3)$. The Horizontal Higgses are taken as a sextet $\chi$ and two triplets $\eta, \xi$, with the VEVs $\sim M_X$. We assume that their VEVs have the structure [16], which can be indeed obtained in analysis of their superpotential [17]: $\langle \Sigma \rangle = M \text{diag}(1, 1, -2)$, $M \geq A, B, C$. All these are singlets of $SU(5) \times SU(5)'$.

We assume further that the only fermion states present at the scale $M$ besides the chiral $f$ fermions $\bar 5 + 10$, are the states $X^\alpha + \bar X^\alpha$, which acquire mass via $\Sigma$, and by some symmetry reason only the Higgses $\chi$ and $H$ couple these fermions in a renormalizable way:

\[
10\chi \bar X + \bar X \Sigma X + gX10H
\]  

while other Higgses interact via higher order operators:

\[
\frac{10\Omega_2 \Omega_2}{\Lambda^2} \eta \bar X + \frac{10\Omega_2 \Omega_2 \Omega_4 \Omega_4}{\Lambda^4} \xi \bar X + g'X5 \Omega_2 \bar H'
\]  

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where $\Lambda \gg M$ is some cutoff scale, say $\Lambda \sim M_1$. Needless to say that these operators can be effectively induced via integrating out the $F$-states with masses $\sim \Lambda$ along the lines discussed at the end of previous section, so that they do not introduce the flavour violating effects.

Then, by taking into account the VEV pattern (1), from eqs. (12) in the seesaw approximation we arrive to the following textures:

$$
\hat{\lambda}_u = \frac{g}{M} \left( \begin{array}{ccc}
0 & \kappa_u A & 0 \\
-\kappa_u A & 0 & \frac{1}{2} \varepsilon_u B \\
0 & \varepsilon_u B & C
\end{array} \right)
$$

$$
\hat{\lambda}_d = \frac{\varepsilon g'}{M} \left( \begin{array}{ccc}
0 & \kappa_d A & 0 \\
-\kappa_d A & 0 & \frac{1}{2} \varepsilon_d B \\
0 & \varepsilon_d B & \frac{1}{2} C
\end{array} \right)
$$

$$
\hat{\lambda}_e = \frac{\varepsilon g'}{M} \left( \begin{array}{ccc}
0 & -\kappa_e A & 0 \\
\kappa_e A & 0 & \varepsilon_e B \\
0 & \varepsilon_e B & \frac{1}{2} C
\end{array} \right)
$$

(22)

where $\varepsilon = M_2 / \Lambda$, and $\varepsilon_{u,d,e} \sim \varepsilon^2$, $\kappa_{u,d,e} \sim \varepsilon^4$ contain Clebsch factors emerging from the first two term in (21). In particular, the structure of the VEV $\Omega_2$ leads to $\varepsilon_u = \varepsilon_d = \frac{1}{2} \varepsilon_e$, while $\kappa_{u,d,e}$ in general are not related due to complex structure of the $\Omega_4$ and $\bar{\Omega}_4$ VEVs.

The fact that $H_1$ couples to fermions via higher order operator, allows us to assume that the constant $g'$ is order 1 as well as the Yukawa constant $g \sim \lambda_t$, and at the same time to have small $\tan \beta$. The smallness of the bottom-tau masses with respect to top in this case can be attributed to the suppression factor $\varepsilon = M_2 / \Lambda$ rather than to large $\tan \beta$. On the other hand, the structure of $\langle \Omega_2 \rangle$ implies that colour triplet $\bar{T}^i$ in $\bar{H}'$ is decoupled from $\bar{5}$ states and thus proton is perfectly stable in this theory. Interesting enough, eq. (13) indicates that the $\bar{u}_c$ mass matrix $M_{\bar{u}_c}^2$ is degenerated between first two families, which provides further suppression of the flavour changing and CP-violating phenomena in the up sector as compared to general case of sect. 2.

The mass matrix texture, which depends on 7 real parameters, allows to make predictions. In a leading approximation, we obtain the following relations between the MSSM Yukawa constants at the GUT scale:

$$
\lambda_b = \lambda_\tau, \quad \frac{\lambda_c}{\lambda_t} = \frac{1}{4} \frac{\lambda_s}{\lambda_t} = \frac{1}{16} \frac{\lambda_u}{\lambda_\tau}
$$

(23)

and the expressions for the CKM mixing angles

$$
\sin 12 = \sqrt{\frac{\lambda_d}{\lambda_s} - e^{i \delta} \sqrt{\frac{\lambda_u}{\lambda_c}}},
$$

$$
\sin 23 = \sqrt{\frac{\lambda_b}{2 \lambda_b} - \frac{\lambda_s}{2 \lambda_t}} = \frac{1}{4} \sqrt{\frac{\lambda_b}{2 \lambda_t}} = 0.043
$$

$$
\frac{v_{ab}}{v_{cb}} = \sqrt{\frac{\lambda_u}{\lambda_c}} \quad \frac{|V_{td}|}{|V_{ts}|} = \sqrt{\frac{\lambda_d}{\lambda_s}}
$$

(24)
where $\delta$ is a CP-violating phase. In particular, when $\delta \sim 1$, we obtain $s_{12} \approx \sqrt{m_d/m_s}$.

The exact implications of these GUT scale relations for the low energy observables, though almost obvious, will be given elsewhere.

Let us conclude this section with the following remark. The seesaw limit is certainly good for first two families. However, the fact that $\lambda_t \sim 1$ implies for the 33 entry of matrices $\hat{\lambda}_{u,d,e}/M \sim 1$ unless the coupling constant $g$ is much above the perturbativity bound. Thus for the Yukawa constants of the third generation one has to use the exact formula (see e.g. in [18]). Then the genuine Yukawa constants of the third family $\tilde{\lambda}_{t,b,\tau}$ are related to the ‘would-be’ Yukawa constants $\lambda_{t,b,\tau}$ given in the seesaw limit (12) as

$$\tilde{\lambda}_{t,b,\tau} = \frac{\lambda_{t,b,\tau}}{\sqrt{1 + (\lambda_t/2g)^2}} < \lambda_{t,b,\tau} \quad (25)$$

Therefore, for $\lambda_t \simeq 1$, as it is favoured from eqs. (23), implies that seesaw limit can be reasonably good already for $g = 1$.

To conclude, it seems that in order to deal with the fermion flavour and sflavour problems in a coherent way, All you need is LOVE (SUSY+GUT), though with a little help of little friend [19] which comes from the horizontally understood symmetry properties.

References


R. Barbieri, G. Dvali and L.J. Hall, LBL-38065 (1996); 


