Higgs- and quark-inspired modifications of the finite-temperature properties of the Polyakov model

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(2+1)-dimensional Georgi-Glashow model, else called the Polyakov model, is explored at nonzero temperatures and in the regime when the Higgs boson is not infinitely heavy. The finiteness of the Higgs-boson mass leads to the appearance of the upper bound on the parameter of the weak-coupling approximation, necessary to maintain the stochasticity of the Higgs vacuum. The modification of the finite-temperature behavior of the model emerging due to the introduction of massless quarks is also discussed.

1. Introduction. The model

Since the second half of the seventies [1], (2+1)-dimensional Georgi-Glashow model, oftenly also called the Polyakov model, is known as an example of the theory allowing for an analytic description of confinement. However, confinement in the Polyakov model is typically discussed in the limit of infinitely large Higgs-boson mass, when the model is reduced to compact QED. In the present talk, we shall discuss various modifications of the finite-temperature properties of the Polyakov model stemming from the finiteness of this mass.

The Euclidean action of the model reads

\[ S = \int d^3x \left[ \frac{1}{4g^2} (F_{\mu\nu}^a)^2 + \frac{1}{2} (D_\mu \Phi^a)^2 + \frac{\lambda}{4} (\Phi^a)^2 - \eta^2 \right] . \]  

Here, the Higgs field \( \Phi^a \) transforms by the adjoint representation, \( D_\mu \Phi^a \equiv \partial_\mu \Phi^a + \epsilon^{abc} A_\mu^b \Phi^c \). Next, \( \lambda \) is the Higgs coupling constant of dimensionality [mass], \( \eta \) is the Higgs v.e.v. of dimensionality [mass]^{1/2}, and \( g \) is the electric coupling constant of the same dimensionality. At the one-loop level, the sector of the theory [1] containing dual photons and Higgs bosons is represented by the following partition function [2]:

\[ Z = 1 + \sum_{N=1}^{\infty} \frac{\zeta^N}{N!} \left( \prod_{i=1}^{N} \int d^3z_i \sum_{q_i=\pm 1} \right) \exp \left[ -\frac{g_m^2}{8\pi} \times \right. \]

\[ \times \sum_{n, k=1}^{N} \left( \frac{q_n q_k}{|z_n - z_k|} - e^{-m_H |z_n - z_k|} \right) \]

\[ \equiv \int D\chi D\psi e^{-S}, \]  

where

\[ S = \int d^3x \left[ \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} (\partial_\mu \psi)^2 + \frac{m_H^2}{2} \psi^2 - 2\zeta e^{g_m \psi} \cos(g_m \chi) \right] \equiv \int d^3x L[\chi, \psi |g_m, \zeta]. \]  

The partition function [2] describes the grand canonical ensemble of monopoles with the account for their Higgs-mediated interaction. In Eqs. [2] and [3], \( \chi \) is the dual-photon field, and the field \( \psi \) accounts for the Higgs field, whose mass reads \( m_H = \eta \sqrt{2\lambda} \). Note that from Eq. [2] it is straightforward to deduce that when \( m_H \) formally tends to infinity, one arrives at the conventional sine-Gordon theory of the dual-photon field [1] describing the compact-QED
limit of the model. Next, in the above equations, \( g_m \) stands for the magnetic coupling constant related to the electric one as \( g_m g = 4\pi \), and the monopole fugacity \( \zeta \) has the form \( \zeta = m_H^2 g \sqrt{\frac{\lambda}{2\pi}} e^{-4\pi m_W \epsilon / g^2} \). In this formula, \( m_W = g\eta \) is the W-boson mass, and \( \epsilon = \epsilon(\lambda / g^2) \) is a certain monotonic, slowly varying function, \( \epsilon \geq 1 \), \( \epsilon(0) = 1 \) [3], \( \epsilon(\infty) \approx 1.787 \) [4]. As far as the function \( \delta \) is concerned, it is determined by the loop corrections. In what follows, we shall work in the standard weak-coupling regime \( g^2 \ll m_W \), which parallels the requirement that \( \eta \) should be large enough to ensure the spontaneous symmetry breaking from \( SU(2) \) to \( U(1) \). The W-boson mass will thus play the role of the UV cutoff in the further analysis.

2. The model at finite temperature beyond the compact-QED limit

In the discussion of finite-temperature properties of the Polyakov model in the present Section, we shall follow Ref. [5]. At finite temperature \( T \equiv 1/\beta \), one should supply the fields \( \chi \) and \( \psi \) with the periodic boundary conditions in the temporal direction, with the period equal to \( \beta \). Because of that, the lines of magnetic field emitted by a monopole cannot cross the boundary of the one-period region and consequently, at the distances larger than \( \beta \), should go almost parallel to this boundary, approaching it. Therefore, monopoles separated by such distances interact via the 2D Coulomb potential, rather than the 3D one. Since the average distance between monopoles in the plasma is of the order \( \zeta^{-1/3} \), we see that at \( T \geq \mathcal{O}(\zeta^{-1/3}) \), the monopole ensemble becomes two-dimensional. Owing to the fact that \( \zeta \) is exponentially small in the weak-coupling regime under discussion, the idea of dimensional reduction is perfectly applicable at the temperatures of the order of the critical temperature of the Berezinsky-Kosterlitz-Thouless (BKT) phase transition [5] (for a review see e.g. Ref. [7]) in the monopole plasma, which is equal to \( g^2/2\pi \) [8] 1.

Up to exponentially small corrections, this temperature is unaffected by the finiteness of the Higgs-boson mass. This can be seen from the expression for the mean squared separation in the monopole-antimonopole molecule,

\[
\langle L^2 \rangle = \frac{\int d^2 x \langle |x| > m_W^2 \rangle d^2 \sqrt{2} \frac{2\pi T}{2\pi} J}{\int d^2 x \langle |x| > m_W^2 \rangle d^2 \sqrt{2} \frac{2\pi T}{2\pi} J},
\]

where \( J = \exp \left[ \frac{4\pi T}{2\pi} K_0 (m_H |x|) \right] \) and \( K_0 \) denotes the modified Bessel function. Disregarding the exponential factors in the numerator and denominator of this equation, we obtain \( \langle L^2 \rangle \approx \frac{2\pi T}{2\pi} \), that yields the above-mentioned value of the BKT critical temperature \( g^2/2\pi \). Besides that, it is straightforward to see that in the weak-coupling regime under study, the value of \( \sqrt{\langle L^2 \rangle} \) is exponentially smaller than the characteristic distance in the monopole plasma, \( \zeta^{-1/3} \), i.e., molecules are very small-sized with respect to that distance.

The factor \( \beta \) at the action of the dimensionally-reduced theory, \( S_{\text{d.-r.}} = \beta \int d^2 x \mathcal{L}[\chi, \psi | g_m, \zeta] \), can be removed [and this action can be cast to the original form of eq. (3) with the substitution \( d^3 x \rightarrow d^2 x \)] by the obvious rescaling: \( S_{\text{d.-r.}} = \int d^2 x \mathcal{L} \left[ \chi^{\text{new}}, \psi^{\text{new}} | \sqrt{K}, \beta \zeta \right] \). Here, \( K \equiv g_m^2 T, \chi^{\text{new}} = \sqrt{3} \chi, \psi^{\text{new}} = \sqrt{\beta} \psi \), and in what follows we shall denote for brevity \( \chi^{\text{new}} \) and \( \psi^{\text{new}} \) simply as \( \chi \) and \( \psi \), respectively. Averaging then over the field \( \psi \) with the use of the cumulant expansion we arrive at the following action:

\[
S_{\text{d.-r.}} \simeq \int d^2 x \left[ \frac{1}{2}(\nabla \chi)^2 - 2\xi \cos \left( g_m \sqrt{T} \chi \right) \right] - 2\xi^2 \int d^2 x d^2 y \cos \left( \sqrt{K} \chi(x) \right) K^{(2)}(x - y) \times \cos \left( \sqrt{K} \chi(y) \right),
\]
In this expression, we have disregarded all the cumulants higher than the quadratic one, and the limits of applicability of this so-called bical approximation will be discussed below. Further, in Eq. \( \mathbb{K}(x) \equiv e^{K_{m_{h}}^{(2)}(x)} - 1 \), where \( D_{m_{h}}^{(2)}(x) \equiv K_{0}(m_{h}|x|)/2\pi \) is the 2D Yukawa propagator, and \( \xi \equiv \beta \xi_{e} D_{m_{h}}^{(2)}(m_{W}^{-1}) \) denotes the monopole fugacity modified by the interaction of monopoles via the Higgs field. Clearly, in the compact-QED limit (when \( m_{h} \) formally tends to infinity) \( D_{m_{h}}^{(2)}(m_{W}^{-1}) \to 0 \), and \( \xi \to \beta \xi_{e} \), as it should be. In the general case, when the mass of the Higgs field is moderate and does not exceed \( m_{W} \), we obtain \( \xi \propto \exp \left[ -\frac{2\xi}{\pi} \left( m_{W} \epsilon + T \ln \left( \frac{\xi}{c} \right) \right) \right] \). Here, we have introduced the notation \( c \equiv m_{h}/m_{W} \), \( c < 1 \), and \( \gamma \approx 0.577 \) is the Euler constant, so that \( \frac{2\xi}{\pi} \approx 0.89 < 1 \). We see that the modified fugacity remains exponentially small, provided that

\[
T < -\frac{m_{W} \epsilon}{\ln \left( \frac{\xi}{c} \right)}.
\]

This constraint should be updated by another one, which would provide the convergence of the cumulant expansion applied in course of the average over \( \psi \). Were the cumulant expansion divergent, this fact would indicate that the Higgs vacuum loses its normal stochastic properties and becomes a coherent one. In order to get the new constraint, notice that the parameter of the cumulant expansion reads \( \xi I^{(2)} \), where \( I^{(2)} \equiv \int d^{2}x \mathbb{K}^{(2)}(x) \). The most essential, exponential, part of the parameter of the cumulant expansion then reads \[ \xi I^{(2)} \propto \exp \left[ -\frac{2\xi}{\pi} \left( m_{W} \epsilon + T \ln \left( \frac{\xi}{c} \right) - T \frac{\gamma e^{\gamma}}{e^{\gamma} - 1} \right) \right] \]. Therefore, the cumulant expansion converges at the temperatures obeying the inequality

\[
T < -\frac{m_{W} \epsilon}{\frac{2\xi}{\pi} - \ln \left( \frac{\xi}{c} \right)},
\]

which updates the inequality \[ m_{W} \epsilon < 2\pi \epsilon \]

Note that although this inequality is satisfied automatically at \( \frac{2\xi}{\pi} \approx 1 \), since it then takes the form \( m_{W} \epsilon < \sqrt{2\pi} \epsilon \), this is not so for the Bogomolny-Prasad-Sommerfield limit, \( c \ll 1 \). Indeed, in such a case, we have \( \frac{2\xi}{\pi} \ln \left( \frac{\xi}{c} \right) < 2\pi \epsilon \), that owing to the logarithm is however quite feasible.

### 3. Including massless quarks

Let us consider the extension of the model \[ \mathbb{K}(x) \] by the fundamental dynamical quarks, which are supposed to be massless: \( \Delta S = -i \int d^{3}x \bar{\psi} \gamma \sigma D \psi \). In this formula, \( D_{\mu} \psi = (\partial_{\mu} - ig_{A_{\mu}}^{\gamma} A_{\gamma}) \psi, \bar{\psi} = \psi^{\dagger} \beta \), where the Euclidean Dirac matrices are defined as \( \bar{\gamma} = -i \beta \bar{\alpha} \) with \( \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \) and \( \bar{\alpha} = \begin{pmatrix} 0 & \varphi \\ -\varphi & 0 \end{pmatrix} \). Our discussion in the present Section will further follow Ref. \[ \mathbb{K} \]. In that paper, it has been shown that at the temperatures higher than the BKT one, quark zero modes in the monopole field lead to the additional attraction between a monopole and an antimonopole in the molecule. In particular, when the number of these modes (equal to the number of massless flavors) is sufficiently large, the molecule shrinks so strongly that its size becomes of the order of the inverse W-boson mass. Another factor which determines the size of the molecule is the characteristic range of localization of zero modes. Namely, it can be shown that the stronger zero modes are localized in the vicinity of the monopole center, the smaller molecular size is. Let us consider the case when the Yukawa coupling of quarks with the Higgs field vanishes, and originally massless quarks do not acquire any mass. This means that zero modes are maximally delocalized. We shall see that in the case of one flavor, such a weakness of the quark-mediated interaction of monopoles opens a possibility for molecules to undergo the phase transition into the plasma phase at the temperature comparable with the BKT one.

It is a well known fact that in 3D, \('t Hooft-
Polyakov monopole is actually an instanton [1]. Owing to this fact, we can use the results of Ref. [10] on the quark contribution to the effective action of the instanton-antiinstanton molecule in QCD. Referring the reader for details to Ref. [9], we shall present here the final expression for the effective action, which reads \( \Gamma = 2N_f \ln |a| \). Here, \( a = \langle \psi_0^M | g^{i \vec{a} \cdot \vec{a} M} | \psi_0^M \rangle \) is the matrix element of the monopole field \( \vec{a} M \) taken between the zero modes \( | \psi_0^M \rangle \) of the operator \(-i \vec{a} \cdot \vec{D}\) defined at the field of a monopole and an antimonopole, respectively. The dependence of \(|a|\) on the molecular size \( R \) can be straightforwardly found and reads \(|a| \propto \int d^3 r / (\pi^2 R^2 - \vec{R}^2) \approx -4\pi \ln (\mu R)\), where \( \mu \) stands for the IR cutoff.

At finite temperature, in the dimensionally-reduced theory, the usual Coulomb interaction of monopoles goes over to \(-2T \ln (\mu R)\), where \( R \) denotes the absolute value of the 2D vector \( \vec{R} \).

As far as the novel logarithmic interaction, \( \ln (\mu R) = \sum_{n=-\infty}^{+\infty} \ln \left[ \mu (R^2 + (\beta n)^2)^{1/2} \right] \), is concerned, it transforms into \( \pi T R + \ln [1 - \exp(-2\pi T R)] - \ln 2 \), and reads \(|a| \propto \int d^3 r / (\pi^2 R^2 - \vec{R}^2) \approx -4\pi \ln (\mu R)\), where \( \mu \) stands for the IR cutoff.

At large \( R \), \( \ln 2 \& \ln [1 - \exp(-2\pi T R)] \ll \pi T R \).

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Hence, we see that \( \langle L^2 \rangle \) is finite at \( T > (2 - N_f)g^2/4\pi \), that reproduces the standard result \( g^2 / 2\pi \) at \( N_f = 0 \). For \( N_f = 1 \), the plasma phase is still present at \( T < g^2 / 4\pi \), whereas for \( N_f \geq 2 \) the monopole ensemble exists only in the molecular phase at any temperature larger than \( c^{1/3} \). Clearly, at \( N_f \gg \max \{1, 4\pi T / g^2\} \), \( \sqrt{\langle L^2 \rangle} \rightarrow m_\pi^{-1} \), i.e., such a large number of zero modes shrinks the molecule to the minimal admissible size.

**Acknowledgments**

The author is grateful to Dr. N.O. Agasian for collaboration and to Prof. A. Di Giacomo for useful discussions. This work has been supported by INFN and partially by the INTAS grant Open Call 2000, Project No. 110. And last but not least, the author is grateful to the organizers of the International Conference “QCD02” (Montpellier, 2-9th July 2002) for an opportunity to present these results in a very pleasant and stimulating atmosphere.

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