Reaction Mechanisms with Exotic Nuclei*

Angela Bonaccorso †
Istituto Nazionale di Fisica Nucleare, Sez. di Pisa,
56127 Pisa, Italy

November 11, 2013

Abstract

This talk examines a number of reaction mechanisms for scattering initiated by an exotic projectile. Comparisons are made with recent experimental data, in order to extract information on the peculiarity of the nuclear structure under extreme conditions and to test the accuracy of the available theoretical methods. Predictions for future experiments are also made.

1 Introduction

Nuclei far from the stability valley are often called "exotic" because they exhibit properties rather different from those of nuclei in the rest of the nuclear chart [1]. Most of them are neutron rich and unstable against $\beta$-decay. It is interesting to study them because they give information on the structure of matter under extreme conditions and allow to test nuclear models otherwise based only on properties of stable nuclei.

From the theoretical point of view the most interesting characteristic of medium-light unstable nuclei is the fact that single particle

---

∗Invited talk given at the Symposium on Nuclear Clusters, Rauschholzhausen, Germany, 5-9 August 2002.
†In collaboration with G. Blanchon, D.M. Brink, J. Margueron, N. Vinh Mau.
degrees of freedom dominate both in the structure description as well as in the reaction studies. So far the most studied cases have been those of light nuclei like $^6\text{He}$, $^{11}\text{Be}$ $^{11}\text{Li}$ which exhibit the so called halo. $^{19}\text{C}$ is another interesting candidate still under investigation [2, 3, 4]. Due to the fact that the last neutron or couple of neutrons (in $^6\text{He}$ and $^{11}\text{Li}$), are weakly bound, with a separation energy of around 0.5MeV, the wave functions of such valence neutrons exhibit long tails which extend well outside the nuclear potential well and a large part of the single particle strength is already in the continuum. From the structure point of view the dominant feature is the appearance of intruder states in the single particle level scheme and the strong coupling between deformed cores and valence particles. New techniques are needed to study these nuclei, which combine and unify the traditional treatment of bound and continuum scattering states. In this respective reaction theories like the transfer to the continuum [5, 6, 7] can be very useful.

Therefore, as in the early stages of Nuclear Physics, research on light exotic nuclei has concentrated on studying elastic scattering [8]-[17] and spectroscopic properties like the determination of single particle state energies, angular momenta and spectroscopic factors [18, 19, 20].

2 Reaction models for structure studies

There are several cross sections that are measured and calculated in the models. Elastic scattering angular distributions have been measured. The comparison between the scattering of a halo nucleus of mass A with that of the nucleus (A-1) has shown a considerable depletion of cross section which has been explained as due to the breakup channel [8]. On the other hand early measurements concentrated on total reaction cross sections for the extraction of nuclear radii [21, 22] and were also used to disentangle the single particle level sequence of halo nuclei [23].

The next simplest measurement is the single-neutron removal cross section, in which only the projectile residue, namely the core with one less nucleon, is observed in the final state. This information together with the calculated cross sections [2, 3, 18, 19, 20] has been used to extract single particle spectroscopic factors as in traditional transfer
reactions. Besides the integrated removal cross section, denoted by \( \sigma_{-n} \), the differential momentum distribution \( d^3\sigma/dk^3 \) is also measured. A particularly useful cross section is \( d\sigma/dk_z \), the removal cross section differential in longitudinal momentum. It has been used to determine the angular momentum and spin of the neutron initial state \[20\] in a way similar to that proposed in \[7, 24\]. If the final state neutron can also be measured, the corresponding coincident cross section \( A_p \rightarrow (A_p - 1) + n \) is called the diffractive (or elastic) breakup cross section if the interaction responsible for the removal is the neutron-target nuclear potential \[25, 26\]. In the case of heavy targets the coincident cross section contains also the contribution from Coulomb breakup due to the core-target Coulomb potential which acts as an effective force on the neutron. This observable is very useful to disentangle the reaction mechanism \[27\]. The difference between the removal and coincident cross sections is called the stripping (or absorption) cross section.

All theoretical methods used so far rely on a basic approximation to describe the collision with only the three-body variables of nucleon coordinate, projectile coordinate, and target coordinate. Thus the dynamics is controlled by the three potentials describing nucleon-core, nucleon-target, and core-target interactions. In most cases the projectile-target relative motion is treated semiclassically by using a trajectory of the center of the projectile relative to the center of the target \( R(t) = b_c + vt \) with constant velocity \( v \) in the \( z \) direction and impact parameter \( b_c \) in the \( xy \) plane.

### 2.1 Nuclear-Coulomb elastic breakup.

A full description of the treatment of the scattering equation for a projectile which decays by single neutron breakup following its interaction with the target, can be found in \[6, 27\]. There it was shown that within the semiclassical approach for the projectile-target relative motion, the amplitude for a transition from a nucleon bound state \( \psi_i \) in the projectile to a final continuum state \( \psi_f \) is given by

\[
A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt < \psi_f(t)|V(r)\psi_i(t) >=, \tag{1}
\]

where \( V \) is the interaction responsible for the neutron transition to the continuum.
For light targets the recoil effect due to the projectile-target Coulomb potential can be neglected and the interaction responsible for the reaction is mainly the neutron-target nuclear potential. In the case of heavy targets the dominant reaction mechanism is Coulomb breakup. The Coulomb force does not act directly on the neutron but it affects it indirectly by causing the recoil of the charged core. Therefore the neutron is subject to an effective force which gives rise to an effective dipole Coulomb potential \( V_{\text{eff}}(\mathbf{r}, \mathbf{R}(t)) \). In ref. [27] it was shown that the combined effect of the nuclear and Coulomb interactions to all orders can be taken into account by using the potential \( V = V_{\text{nt}} + V_{\text{eff}} \) sum of the neutron-target optical potential and the Coulomb dipole potential. If for the neutron final continuum wave function we take a distorted wave of the eikonal-type, then the amplitude becomes:

\[
A_{fi}(k, b_c) = \frac{1}{i\hbar} \int d^3r \int dt e^{-i(k-r)t+i\omega t} e^{i\int_t^\infty V(\mathbf{r}, t') dt'} V(\mathbf{r}, t) \phi_{l,m_i}(\mathbf{r})
\]

(2)

where \( \omega = (\varepsilon_f' - \varepsilon_0)/\hbar \) and \( \varepsilon_0 \) is the neutron initial bound state energy while \( \varepsilon_f' \) is the neutron-core final continuum energy. Eq. (2) is appropriate to calculate the coincidence cross sections \( A_p \to (A_p-1) + n \) discussed in the previous section. Finally the differential probability with respect to the neutron energy and angles can be written as

\[
\frac{d^3P_{nc}(b_c)}{d\varepsilon_f' \sin \theta d\phi} = \frac{1}{8\pi^3} \frac{mk_n}{\hbar^2} \frac{1}{2l+1} \sum_{m_i} |A_{fi}|^2.
\]

and we have averaged over the neutron initial state.

The effects associated with the core-target interaction will be included by multiplying the above probability by \( P_{ct}(b_c) = |S_{ct}|^2 \) [7] the probability for the core to be left in its ground state, defined in terms of a core-target S-matrix function of \( b_c \), the core-target distance of closest approach. A simple parameterization is \( P_{ct}(b_c) = e^{-\ln 2 \exp[(R_s-b_c)/a]} \), where the strong absorption radius \( R_s \approx 1.4(A_p^{1/3} + A_t^{1/3}) fm \) is defined as the distance of closest approach for a trajectory that is 50% absorbed from the elastic channel and \( a = 0.6 fm \) is a diffusness parameter.

Thus the double differential cross section is

\[
\frac{d^2\sigma}{d\varepsilon_f' d\Omega} = C^2 S \int_0^\infty db_c \frac{d^2P_{nc}(k, b_c)}{d\varepsilon_f' d\Omega} P_{ct}(b_c),
\]

(3)

(see Eq. (2.3) of [7]) and \( C^2 S \) is the spectroscopic factor for the initial single particle orbital.
2.2 Nuclear elastic and absorptive breakup.

Inclusive cross sections in which only the core with \((A_p - 1)\) nucleons is detected need to take into account also the absorption of the neutron by the imaginary part of the n-target optical potential. For such reactions the Coulomb recoil effect can be neglected but the distorted eikonal-type wave function used in Eq. (2) is not accurate enough, in particular if the final continuum states are single particle resonances in the target plus one neutron nucleus. Then a distorted final neutron wave function, calculated by an optical model will be used. Also since the neutron is not detected one integrates over the neutron angles. Thus, according to [6] the final neutron probability energy spectrum with respect to the target reads

\[
\frac{dP}{d\varepsilon_f} \approx \frac{1}{2} \Sigma_j \left| 1 - |\langle S_{jf} \rangle| \right|^2 + 1 - |\langle S_{jf} \rangle|^2 \left( 2j_f + 1 \right) \left( 1 + F_{l_f, i, j_f, j_i} \right) B_{l_f, l_i}.
\]

(4)

where \(S_{jf}\) is the neutron-target optical model S-matrix, \(F_{l_f, i, j_f, j_i}\) is an \(l\) to \(j\) recoupling factor, \(\eta\) is the transverse component of the neutron momentum which is conserved in the neutron transition, \(b_c\) is the core-target impact parameter, \(C_i\) is the initial state asymptotic normalization constant and \(M_{l_f, l_i}\) is a factor depending on the angular parts of the initial and final wave functions, \(v\) is the relative motion velocity at the distance of closest approach.

3 Applications

We are going to discuss now a series of calculations aimed at extracting spectroscopic information on one-neutron and two-neutron halo nuclei.

3.1 Neutron energy distributions in Coulomb breakup.

\(^{11}\)Be is probably the best known one-neutron halo nucleus since experimental information has been available for long time [28]. The ground state is a \(2s_{1/2}\) state with separation energy of 0.5MeV and spectroscopic factor \(C^2S = 0.77\). Therefore it has been used as a test
case for reaction models which use the above basic structure information as input. A very comprehensive study of one neutron breakup mechanism, cross section and momentum distributions can be found in [18, 25, 26, 29, 30].

On the other hand a more recent work [27] has improved the previous knowledge of the breakup reaction, by studying the Coulomb-nuclear interference effects according to Eq.(2). We would like to report here on new calculations that we have recently performed by using Eq.(2) to study higher order effects. We have tested three limits of Eq.(2). The first is the sudden approximation in which $\omega = 0$ and Eq.(2) can be calculated with nuclear and Coulomb to all orders. We call the corresponding amplitude $A_{sudd}^{all-ord}$. Then we have studied the first order approximation for the Coulomb term in which $\omega t \neq 0$ but the $\omega t$ term is kept (this is the standard first order perturbation theory amplitude $A_{pert}^{1st}$) and finally the sudden approximation restricted to first order giving $A_{sudd}^{pert}$. The main results of our new calculations are shown in Fig.(1a) and (1b) which

Figure 1: Neutron-core final energy distribution after nuclear-Coulomb breakup.
give the neutron final energy spectrum with respect to the core for breakup of $^{11}Be$ and $^{19}C$ on $^{208}Pb$ at 72 A.MeV and 67 A.MeV respectively. Experimental data are from [3]. Preliminary calculations indicate that for $R_s < b_c < 50 \text{fm}$ the results obtained with $A_{\text{pert}}$ are equal to those obtained with $A_{\text{sudd}}$, thus showing that at small impact parameters the sudden approximation is valid but higher order terms need to be considered. On the other hand for $b_c > 50 \text{fm}$ we find that higher order effects are negligible since using $A_{\text{sudd}}^{\text{all-ord}}$ or $A_{\text{sudd}}^{\text{pert}}$ does not give any difference. Then we can conclude that at large $b_c$ perturbation theory is valid. Thus in the figures we give by the dotted curves the results of the simple first order perturbation theory while the solid curves are the all order calculations according to an amplitude defined as $A^{\text{exact}} = A_{\text{sudd}}^{\text{all-ord}} + A_{\text{pert}} - A_{\text{sudd}}^{\text{pert}}$, valid at all core-target impact parameters and not giving rise to divergences in the final integral over impact parameters in Eq. (3). In the case of $^{11}Be$ the theoretical calculations have been multiplied by the known spectroscopic factor, while for $^{19}C$ we have used unity spectroscopic factor and a neutron separation energy for the 2s state of 0.5MeV. As expected, and already shown by other authors the effects of higher order terms are to reduce the peak cross section. Analysis of the type presented in this section have been used to extract spectroscopic factors. From our calculations we would extract $C^2S = 0.70$ for the 2s-state of $^{11}Be$ and $C^2S = 0.65$ for $^{19}C$ assuming in both cases a separation energy of 0.5MeV.

### 3.2 $^{10}Li$ spectrum and $^{11}Li$ properties.

We discuss now the results of a possible reaction aiming at clarifying the structure $^{11}Li$ which has been a challenge for long time [23, 31-40]. This nucleus and $^{6}He$ are two-neutron halo nuclei. They are special because their corresponding A-1 systems are unbound and it is thanks to the pairing force acting between the two neutrons that they become bound. $^{11}Li$ has been very difficult to study from the experimental point of view because the ground state of $^{10}Li$ is unbound [34-40], and one of the available states for the valence neutron is a $2s_{1/2}$ virtual state which does not even have a centrifugal barrier. $^{6}He$ is different in this respect because the $p_{3/2}$ ground state resonance of $^{5}He$ has a width of about 600KeV corresponding to a lifetime of about 300fm/c [11] and its decay in flight has been clearly observed. Recently a pickup experiment $d(^{11}Be, ^{3}He)^{10}Li$ [40] has definitely confirmed the earlier hypothesis that the ground state of
$^{10}$Li is the 2s virtual state and that the $1p_{1/2}$ orbit gives an excited state. Three body models of $^{11}$Li need as a fundamental ingredient the n-core ($^{9}$Li) interaction, which in turn determines the energies of the low energy unbound states in $^{10}$Li. Following ref.[32] the two neutron hamiltonian is $H_{2n} = h_1 + h_2 + V_{nn}$. $V_{nn}$ is the zero-range pairing interaction. The single neutron hamiltonian is $h = t + V_{cn}$ where $t$ is the kinetic energy and $V_{cn} = V_{WS} + \delta V$ is the neutron-core interaction. It is given by the usual Woods-Saxon potential plus spin-orbit plus a correction $\delta V$ which originates from particle-vibration couplings. They are important for low energy states but can be neglected at higher energies. If Bohr and Mottelson collective model is used for the transition amplitudes between zero and one phonon states, then $\delta V(r) = 16\alpha e^{2(r-R)/a}/(1 + e^{(r-R)/a})^4$ where $R \approx r_0 A^{1/3}$. According to [32] the best parameters for the n-$^9$Li interaction given in Table 1. The corresponding energies obtained for the 2s and 1$p_{1/2}$ state are given in Table 2, together with the values of the strength $\alpha$ of the correction potential $\delta V$.
Table 1: Woods-Saxon and spin-orbit potential parameters

<table>
<thead>
<tr>
<th>$V_0$</th>
<th>$r_0$</th>
<th>$a$</th>
<th>$V_{so}$</th>
<th>$a_{so}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MeV)</td>
<td>(fm)</td>
<td>(fm)</td>
<td>(MeV)</td>
<td>(fm)</td>
</tr>
<tr>
<td>39.83</td>
<td>1.27</td>
<td>0.75</td>
<td>7.07</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 2: Energies of the s and p states, width of unbound p-state, scattering length of the s-state and strength parameter for the $\delta V$ potential. (a) bound-state like calculation, (b) scattering state calculation

<table>
<thead>
<tr>
<th>$\epsilon_{2s}(MeV)$</th>
<th>$\Gamma$(MeV)</th>
<th>$a_s$(fm)</th>
<th>$\alpha$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.123</td>
<td>0.17</td>
<td>-48.5</td>
<td>-13.3</td>
</tr>
<tr>
<td>0.45</td>
<td>-20</td>
<td>-14.0</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_{1p_{1/2}}(MeV)$</td>
<td>0.485</td>
<td>0.595</td>
<td>0.48</td>
</tr>
</tbody>
</table>

It would be therefore extremely interesting and important if an experiment could determine the energies of the two unbound $^{10}$Li states such that the interaction parameter could be deduced. Two $^9$Li($d, p)^{10}$Li experiments have recently been performed. One at MSU at 20 A.MeV [42] and the other at the CERN REX-ISOLDE facility at 2 A.MeV [43]. For such transfer to the continuum reactions the theory underlined in Section (2.2) is very accurate. We present in the following our predictions. In order to study the sensitivity of the results on the target and on the energies assumed for the s and p states, we have calculated the reaction $^9$Li($X, X-1)^{10}$Li at 2 A.MeV for three targets $d$, $^9$Be, $^{13}$C. The $^{13}$C target has been chosen because in such a case the neutron transfer to the 2s state in $^9$Li would be a spin-flip transition which as it is well known are enhanced at low incident energy. For the other two cases the transfer to the 2s state is a non spin-flip transition which is hindered. We show in Fig.2 the neutron energy spectrum relative to $^9$Li obtained with the interaction and single particle energies of Tables 1 and 2. In the case of the 2s virtual state we give also the scattering length obtained as $a_s = \lim_{k \to 0} -\frac{\tan \delta_0}{k}$. We define the resonance energy of the p-state and the energy of the virtual s-state as the energy at which $\delta_l = \pi/2$ and therefore $\text{Im}S_l$ changes sign. We
have checked that in this way we get the s continuum state at the same positive energy if we solve the Schröedinger equation in a box or if we solve it with the proper scattering boundary conditions. This is also the energy at which $|1 - \langle S_{jf}\rangle|^2$ in eq. (4) gets its maximum value. It is important to stress such a definition in the case of the s-virtual state. This is because our prescription gives different energies than those obtained using the relation $\varepsilon_{2s} = \frac{h^2}{2ma^2}$. The results of Fig.2 show that the peak of the p-state will determine without ambiguity the position of the p-state in a target independent way. The width instead is modified by the reaction mechanism, but it can however be obtained from the theory (actually from the phase shift behavior) once that the energy is fixed. For the s-state we see that there is a larger probability of population in the spin-flip reaction initiated by the carbon target. A measure of the line-shape (or spectral function) and absolute value of the cross section will determine the energy of the state also in this case. The integral over energy of the energy distribution will determine the spectroscopic factor of the state. In this case there is no spreading of the single particle state since the n-$^9$Li interaction is real at such low energies. In fact the first excited state of $^9$Li is at $E^* = 2.7 MeV$. In order to demonstrate the sensitivity of the model calculation to the energy of the state we show in Fig.2 by the crosses the result obtained if the s-state is located at $\varepsilon_{2s} = 0.45 MeV$ corresponding to $a_s = -20 fm$. In this case a clear peak appears even if located at very small energy. The fact that a peak appears or does not appear in the transfer spectrum, depends on the relative behavior of the two terms $|1 - \langle S\rangle|^2$ and $B(j_i, j_f..)$ in Eq.(4). The $|1 - \langle S\rangle|^2$ term has always a maximum value equal to 4 at the energy of the state, while the B-term has a divergent-like behavior as the energy approaches zero. Therefore we can conclude that if a transfer to the continuum experiment could measure with sufficient accuracy (energy resolution) the line-shapes or energy distribution functions for the s and p-states in $^{10}$Li our theory would be able to fix unambiguously the energies of the states. Those in turn could be used to test microscopic models of the n-$^9$Li interaction.

4 Conclusions and future challenges

It is clear from what we have discussed in this paper that physics with radioactive beams is an extremely fascinating field in which the in-
terplay between the understanding of the nuclear structure and that of the reaction mechanism is very strong and an enormous number of progress has been made in the last few years. There are however a number of improvements both experimental as well as theoretical that need to be pursued. Almost all experiments so far performed have been inclusive with respect to the target. Up to date few experiments with full kinematics reconstruction have been performed such as those of Galin and collaborators [44]. But targets like $^9\text{Be}$ which has been widely used, are themselves very weakly bound and probably undergo breakup following the interaction with radioactive beams. Data presently available most probably contain such contributions. The picture contained in Eq. (3) needs therefore to be modified to take into account more complicated situations in which the core-target scattering is NOT elastic. Spin coupling effects and final neutron energy dependence have been neglected in most of the theoretical approaches. The discussion about $^{34}\text{Si}$ in [20] clearly shows that if we are going to study heavier systems in which binding energies might not be so small to generate halos but rather neutron skins, such effects will need to be taken into account. On the other hand two-neutron halo breakup of $^{11}\text{Li}$ has been treated as a process in which the two neutrons are emitted simultaneously in a single breakup process. This is in fact not correct for the second neutron which decays in flight from a resonant state, as seen for $^6\text{He}$, and therefore cannot be described by a breakup form factor of the same type as for the first neutron.

References


P. Roussel-Chomaz et al. private communication.


Y. Patois, Ph.D. Thesis, University of Caen (2001), unpublished,
J. Galin, private communication.