The solar neutrino problem
after three hundred days of data at SuperKamiokande

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Abstract

We present an updated analysis of the solar neutrino problem in terms of both Mikheyev-Smirnov-Wolfenstein (MSW) and vacuum neutrino oscillations, with the inclusion of the preliminary data collected by the SuperKamiokande experiment during 306.3 days of operation. In particular, the observed energy spectrum of the recoil electrons from $^8$B neutrino scattering is discussed in detail and is used to constrain the mass-mixing parameter space. It is shown that: 1) the small mixing MSW solution is preferred over the large mixing one; 2) the vacuum oscillation solutions are strongly constrained by the energy spectrum measurement; and 3) the detection of a possible semi-annual modulation of the $^8$B $\nu$ flux due to vacuum oscillations should require at least one more year of operation of SuperKamiokande.

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I. INTRODUCTION

The solar neutrino problem \cite{1}, namely, the deficit of the neutrino rates measured by the four pioneering solar neutrino experiments, Homestake \cite{2}, Kamiokande \cite{3}, SAGE \cite{4}, and GALLEX \cite{5}, as compared to the standard solar model predictions \cite{6}, represents one of the most convincing indications for new physics beyond the standard electroweak theory. On the one hand, the confidence in the performances of the four detectors has increased continuously, as a result of many careful experimental cross-checks and calibrations \cite{7–10}. On the other hand, the reliability of the most refined solar evolution models (including light element diffusion) has been corroborated by the impressive agreement with a growing amount of helioseismological data \cite{11,12}. Therefore, new neutrino physics (such as neutrino oscillations) appears to be a likely solution of the solar $\nu$ deficit.

However, the deficit of the neutrino event rate is a solar model dependent quantity. It is highly desirable, instead, to measure observables whose interpretation is not related to a prior knowledge of the absolute neutrino flux, such as shape deviations of energy spectra, or relative variations of the event rates during the time of the year. Such measurements have been pioneered by the Kamiokande experiment \cite{8}, but with statistics too low to provide definitive indications. This experimental program is now being pursued with much higher statistics by the real-time, water-Cherenkov SuperKamiokande experiment \cite{13}.

The preliminary results after the first 306.3 days of operation of SuperKamiokande \cite{14–16} are already sufficiently accurate to provide interesting new insights into the solar neutrino problem, although further data are required to draw decisive conclusions. Therefore, we think it useful to present an updated analysis of the solar neutrino problem, including the most recent results from the four pioneering experiments \cite{17,7,8,18,19} and from the SuperKamiokande experiment after 306.3 days \cite{14–16}, that have become publicly available during the 1997 summer conferences. In particular, we discuss in detail the information provided by the energy spectrum of recoil electrons observed at SuperKamiokande, which is one of the most important solar model independent observables. The available experimental information is interpreted in the light of two-flavor neutrino oscillations, both in vacuum \cite{20} and in matter, according to the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism \cite{21}.

The paper is organized as follows. In Sec. II we analyze the solar neutrino problem using the available solar model dependent information (namely, the absolute neutrino rates). In Sec. III we analyze the solar model independent information, and in particular the electron energy spectrum measured by SuperKamiokande, in order to constrain the oscillation interpretation both in matter and in vacuum. We draw our conclusions in Sec. IV. Some technical aspects of the energy spectrum analysis are elucidated in the Appendix.

II. SOLAR MODEL DEPENDENT INFORMATION

In this section we report the most recent measurements of the solar neutrino rates, and compare them to the expectations of the 1995 Bahcall-Pinsonneault (BP95) standard solar model \cite{6}. The observed deficit is interpreted in terms of MSW and vacuum two-family oscillations in the parameter space spanned by the neutrino squared mass difference $\delta m^2$ and by the mixing angle $\theta$. 

2
A. The solar neutrino deficit

Table I reports the neutrino event rates measured by the four pioneering solar neutrino experiments Homestake [17], Kamiokande [8], SAGE [18], and GALLEX [19], together with the $^8$B neutrino flux measurement after 306.3 days at SuperKamiokande [14–16]. The observed rates appear to be significantly smaller than the theoretical expectations of the BP95 model.

Figure 1 shows graphically the information reported in Table I, with the further inclusion of the correlations of theoretical uncertainties, which have been calculated as in [22]. In Fig. 1 the GALEX and SAGE data have been combined (in quadrature) in a single (Gallium) result. The same has been done for the Kamiokande and SuperKamiokande data (K+SuperK). In the combination, asymmetric errors have been conservatively symmetrized to the largest one. The 99% C.L. experimental and theoretical ellipses appear to be distinctly separated in all the planes charted by any two experiments. Notice that the (strongly correlated) theoretical errors are rather large as compared to the experimental errors. Therefore, as far as the total neutrino rates are concerned, further reductions of the experimental uncertainties are not expected to change significantly the current picture of the solar $\nu$ deficit, while improvements in the theoretical predictions would have a greater impact.

B. MSW and vacuum oscillation fits

As is well known, a viable explanation of the solar neutrino deficit is represented by matter-enhanced (MSW) neutrino oscillations (see, e.g., [23–25]) or, alternatively, by vacuum neutrino oscillations (see, e.g., [26,25]). Assuming for simplicity oscillations between two families, the analysis involves only two parameters, the neutrino squared mass difference $\delta m^2$ and the mixing angle $\theta$. 

Figure 2 shows the results of our $\chi^2$-analysis of the data reported in Table I, assuming MSW oscillations. Notice that this fit includes only the $\nu_e$ deficit data, and not the solar model independent information provided by the energy spectra or the night-day asymmetry (that will be considered separately in Sec. III). The usual small and large mixing angle solutions appear to be slightly more constrained than in previous fits (see, e.g., [24]), mainly as a result of the new SuperKamiokande data. The absolute minimum of $\chi^2 (\chi^2_{\text{min}} = 1.0)$ is reached at $(\delta m^2, \sin^2 2\theta) = (9.6 \times 10^{-6} \text{ eV}^2, 4.8 \times 10^{-3})$; the secondary minimum ($\chi^2_{\text{sec}} = 2.45$) is reached at $(\delta m^2, \sin^2 2\theta) = (1.5 \times 10^{-5} \text{ eV}^2, 0.58)$. The 90, 95, and 99% C.L. allowed regions coincide with the contours at $\chi^2 - \chi^2_{\text{min}} = 4.61, 5.99, \text{ and } 9.21$, respectively.

Figure 3 shows the analogous results for the vacuum oscillation analysis. We find the minimum value $\chi^2_{\text{min}} = 4.0$ at $(\delta m^2, \sin^2 2\theta) = (8.2 \times 10^{-11} \text{ eV}^2, 0.87)$. A comparison of $\chi^2_{\text{min}}$ seems to indicate a preference of the MSW oscillation scenario over the vacuum oscillation one. However, it should be noted that, in vacuum oscillation fits, the value of $\chi^2_{\text{min}}$ is very sensitive to small shifts in the central values of the experimental results and thus it might change significantly with new data (while it turns out to be more stable in MSW fits).

A few technical remarks are in order. In our oscillation analyses, the GALEX and SAGE data have been combined in quadrature. However, the Kamiokande and SuperKamiokande data have been fitted separately, since these two experiments have different thresholds and
energy resolutions. In particular, according to [13], we use for SuperKamiokande a threshold of 6.5 MeV for the total electron energy $E_e$, and a Gaussian energy resolution function with a $1\sigma$ width of $\pm15\%$ at $T = 10$ MeV ($T = E_e - m_e$), scaling as $1/\sqrt{T}$ at different electron kinetic energies. The input neutrino fluxes have been taken from [8], and the corresponding theoretical uncertainties have been included in the $\chi^2$ covariance matrix as in [22]. Asymmetric errors have been conservatively symmetrized to the largest one. The $^8$B neutrino energy spectrum has been taken from [27]. Concerning the neutrino cross sections, we use the updated calculations of [27] for the chlorine experiment, and of [28] for the water-Cherenkov experiments. In the MSW calculations, the neutrino production regions and the electron density profiles have been taken from [6]. The Earth regeneration effect is included analytically as in [29] for each detector latitude. In the vacuum oscillation calculations, the time average over the year is performed through the analytical approach of [30]. In conclusion, Figs. 2 and 3 represent state-of-the-art results in the field of solar neutrino oscillations.

III. SOLAR MODEL INDEPENDENT INFORMATION

In this section we analyze the solar model independent information about the electron energy spectrum and the time variation of the neutrino rate measured by SuperKamiokande. Since these data are already more accurate than the corresponding Kamiokande ones, we do not include the latter in our analysis.

We recall that the SuperKamiokande experiment has collected a total of 4395 solar neutrino events above threshold ($E_e > 6.5$ MeV) during 306.3 days of operation with a fiducial volume of 22.5 kton [14–16], corresponding to an observed rate of 0.6377 events/day/kton. Given the quoted deficit factor of 0.3685 [14] (see also Table I), the expected rate (BP95 model, no oscillation) is equal to 1.730 events/day/kton.

A. Energy spectrum

The SuperKamiokande experiment has measured the energy spectrum of recoil electrons from $\nu$-$e$ scattering. The spectrum is given [14, 15] in the form of a 16-bin histogram, each bin presenting the ratio between the experimental rate and the theoretical rate. Since the theoretical spectrum has not been explicitly presented, we rely upon our calculations, using for SuperKamiokande the same inputs ($^8$B $\nu$ spectrum, cross section, energy resolution and threshold) described in the previous section, and normalizing the results to the total expected rate of 1,730 events/day/kton. The global information about the energy spectrum is reported in Table II and in Fig. 4.

$^1$The $^8$B spectrum shape errors evaluated in [27] have not been included in the fits of Figs. 2 and 3, since their effect is much smaller than the uncertainty in the absolute $^8$B neutrino flux. However, such shape errors will be included in the analysis of the energy spectrum performed in Sec. III and in the Appendix.
Table II shows the relevant spectral data in each of the 16 bins (numbered in the first column). As shown in the second column, all bins have a 0.5 MeV width, with the exception of the last one, which collects all events between 14 and 20 MeV (total energy). The third and fourth column report the average total energy $E_i$ (in MeV) and the expected event rate $n_i$ (in events/day/kton/bin) for each bin, in the absence of oscillations (our calculation). The sum of the entries in the fourth column gives the expected rate of 1.730 events/day/kton. The fifth column reports the ratio $r_i$ between the experimental and theoretical rate in each bin. The upper and lower errors of $r_i$ are given in the following columns. The sixth and seventh columns report the quadratic sum of statistical and uncorrelated systematic errors ($\sigma_i'$). Finally, the correlated systematic errors ($\sigma_i''$) are given in the last two columns. The data in columns 6–9 have been graphically reduced from the plots shown in [14–16] and thus may be subject to slight inaccuracies. The numbers are given with three decimal places just to avoid further errors due to truncation and round-off. The asymmetry between upper and lower errors is mainly due to two slightly different energy calibration procedures [16]. Shifts in the absolute energy scale represent the most important source of systematic uncertainties, as was emphasized earlier in [31,32].

Figure 4 shows the spectral information in a different way. The solid curve represents our calculation of the electron spectrum, using the best-fit $^8$B neutrino spectrum given in [27]. The shape of the $^8$B neutrino spectrum is subject to some uncertainties, that have been carefully evaluated in [27]. Therefore, we have also used the ±3$\sigma$ deviated neutrino spectra reported in [27] to calculate the effect on the electron spectrum (dotted curves). Notice that in Fig. 4 both the central and the ±3$\sigma$ deviated theoretical electron spectra are renormalized to the same total area as the experimental spectrum (0.6377 events/day/kton). The black circles represent the experimental spectrum, with horizontal error bars spanning the bin widths, and vertical error bars representing the 1$\sigma$ total experimental uncertainties (quadratic sum of statistical and systematic errors). It can be seen that the slope of the experimental spectrum is slightly more gentle than the theoretical spectrum, the rate observed at low (high) energies being somewhat suppressed (enhanced) with respect to the expectations. This indication, if confirmed with significantly smaller errors, would represent unmistakable evidence for a new, energy-dependent process (such as flavor oscillations) affecting solar neutrinos along their path to the Earth.

Since the observed spectrum deviates only slightly from the expected one, it is reasonable to try to “summarize” the spectral information in a single parameter, related in some way to the slope deviation. We adopt the approach advocated in Refs. [31,32,29], where the spectral deformations were expanded in a series of deviations of the spectral moments (the mean, the variance, etc.) from their standard values. Given the present experimental uncertainties, it is sufficient to study the deviations of the first moment, namely, of the average kinetic energy $\langle T \rangle$ of the electrons, from its standard (no oscillation) value $\langle T \rangle_0$. This reduction procedure has the practical advantage that a single parameter ($\Delta \langle T \rangle$) is used in the fits, instead of 16 bins with correlated errors.

Since the value of $\langle T \rangle$ has not been reported in [14–16], we have estimated it from the energy spectrum information presented in Table II and in Fig. 4. The reader is referred to the Appendix for a detailed derivation. Our result for the fractional deviation $\langle T \rangle/\langle T \rangle_0 - 1$ is:
\[
\frac{\langle T \rangle - \langle T \rangle_0}{\langle T \rangle_0} \times 100 = 0.99^{+2.52}_{-0.96},
\]

where the errors represent the total uncertainties at 1σ. This result represents the starting point of our analysis of the energy spectra deviations due to oscillations. It can be noticed that the observed value of \( \langle T \rangle \) is about 1σ higher than the expectations, with an upper error larger than the lower one. Therefore, negative deviations of \( \langle T \rangle \) (i.e., negative “tilts” of the spectrum slope) will be much more constrained than positive deviations.

**B. Time variations**

Possible variations of the neutrino rate during the night would represent evidence for the regeneration of \( \nu_e \)'s in the Earth matter, as expected within the MSW scenario (see, e.g., [29] and references therein). The SuperKamiokande experiment has found no such effect within the present uncertainties. In fact, the measured asymmetry between nighttime (\( N \)) and daytime (\( D \)) neutrino rates is

\[
\frac{N - D}{N + D} = 0.017 \pm 0.026{\text{(stat.)}} \pm 0.017{\text{(syst.)}}.
\]

No evidence for variations of the neutrino rate during the time of the year has been found so far [14]. Semiannual modulations of the signal (in addition to the trivial \( 1/L^2 \) geometrical variations) would indicate the presence of neutrino oscillations in vacuum (see, e.g., [30,33] and references therein). A Fourier analysis of the signal expected in SuperKamiokande shows that this experiment is sensitive only to the first harmonic \( f_1 \) which, in the absence of oscillations, is equal in value to the Earth’s orbit eccentricity \( \varepsilon = 0.0167 \) (see [33] for details). The purely statistical error \( \sigma_f \) of the difference \( f_1 - \varepsilon \) is given by [33]

\[
\sigma_f \simeq \sqrt{\frac{N_S + N_B}{2N_S^2}},
\]

where \( N_S \) and \( N_B \) are the number of signal and background events, respectively.

The numbers \( N_S \) and \( N_B \) depend on the cut applied to the fiducial angle in the direction of the sun (\( \theta_{\text{sun}} \)). Using Eq. (3) and the spectrum of \( N_S + N_B \) in terms of \( \cos \theta_{\text{sun}} \) as reported in [14–16], we derive the values of \( \sigma_f \) shown in Table III. The dependence of \( \sigma_f \) on the cuts applied to \( \cos \theta_{\text{sun}} \) turns out to be weak, since the effect of a lower signal tends to be compensated by a better signal-to-background ratio, and vice versa. On the basis of the results reported in Table III, we assume

\[
\sigma_f \simeq \pm 0.020 \text{ (1 yr)}
\]

as the typical statistical error of the first Fourier harmonic, after one year of operation at SuperKamiokande. The errors corresponding to 2 and 4 years of operation are simply obtained with rescaling factors of \( 1/\sqrt{2} \) and \( 1/2 \), respectively (assuming the current background event rate). Possible systematic errors that might affect the determination of \( f_1 \) are not considered here.
C. Implications for oscillations in matter

Figure 5 illustrates the implications of the electron energy spectrum measured by SuperKamiokande [as summarized by the datum of Eq. (1)] for the MSW scenario. The upper panel shows curves at constant values of the fractional deviation of \( \langle T \rangle \) expected in the presence of MSW oscillations. Similar curves were discussed earlier in [29] (with a different energy threshold) and in [32] (without the Earth regeneration effect). Superposed are the small and large mixing angle solutions found in Fig. 2 (at 95% C.L.). The central value in Eq. (1) is intriguingly close to the values of \( \Delta \langle T \rangle / \langle T \rangle \) within the small mixing angle solution, which thus appears slightly favored as compared to the large mixing one. Unfortunately, the errors in Eq. (1) are still large, so that only excluded regions can be meaningfully derived at present. The lower panel shows the regions excluded at 2, 3, and 4 standard deviations by Eq. (1). For instance, in the regions excluded at the 2\( \sigma \) level, the theoretical value of \( \Delta \langle T \rangle / \langle T \rangle \) is either smaller than \(-0.93\%\) or greater than \(+6.05\%\). The excluded regions are similar to those shown by the SuperKamiokande Collaboration [14–16].

Figure 6 illustrates the implications of the night-day asymmetry datum of Eq. 2 for the MSW scenario. The upper panel shows iso-lines of the night-day asymmetry (percentage) expected in the presence of oscillations. The lower panel shows the regions excluded at 2, 3 and 4 standard deviations by the datum of Eq. 2. The lower part of the large angle solution, where the asymmetry is predicted to be \( \sim 10\% \), appears to be disfavored by the experimental results.

We do not attempt here a global \( \chi^2 \)-combination of all the SuperKamiokande data (total rate, energy spectrum, and night-day variations), since these pieces of information are not independent; in fact, they are just different “projections” of the double differential spectrum of events as a function of time and energy. A proper analysis of the whole SuperKamiokande data will be possible when such spectrum and its uncertainties will become available.

In conclusion, the present data seem to favor the small angle solution with respect to the large mixing one, because: 1) The global value of \( \chi^2 \) is lower at the small mixing solution; 2) The energy spectrum shows a slight deviation consistent with small mixing; 3) Part of the large mixing angle solution is disfavored by the nonobservation of a large night-day asymmetry.

D. Implications for oscillations in vacuum

Figure 7 illustrates the implications the energy spectrum measurement [as summarized by the datum in Eq. (1)] for the vacuum oscillation scenario. The upper panel shows curves at constant values of the fractional deviation of \( \langle T \rangle \) expected in the presence of vacuum oscillations. Similar curves were discussed earlier in [32] (with a different energy threshold for SuperKamiokande). Superposed are the multiple solutions found in the vacuum oscillation fit of Fig. 3 (at 95% C.L.). The solutions at “low \( \delta m^2 \)” (\( \delta m^2 \approx 6–7.5 \times 10^{-11} \text{eV}^2 \)) predict positive deviations of \( \langle T \rangle \), in agreement with the datum of Eq. (1). The “high \( \delta m^2 \)” solutions predict negative deviations, and thus are strongly disfavored, as shown in the lower panel of Fig. 7. Similar results have been discussed by the SuperKamiokande Collaboration [14–16], and are consistent with earlier Kamiokande spectrum fits [23].
Figure 8 illustrates the sensitivity of the SuperKamiokande experiment to semiannual modulations of the neutrino rate induced by vacuum oscillations. The solid curve represents the deviations of the first Fourier coefficient $f_1$ from its standard value $\varepsilon$ (see [33]), plotted as function of $\delta m^2$ in the most favorable condition (maximal oscillation amplitude, $\sin^2 2\theta = 1$). The largest vertical error bar corresponds to the $\pm 1\sigma$ statistical error estimated in Eq. (1) for one year of operation of SuperKamiokande. It can be seen that the 1 yr error bars are as large as the range spanned by variations of $f_1 - \varepsilon$, which are thus practically undetectable at present. However, the error bars after 2 and 4 years of data taking should be reduced enough to allow some sensitivity to the largest predicted variations. Of course, improvements in the energy threshold or in the background rate might reduce the uncertainties more rapidly than the time sequence shown in Fig. 8.

By comparing Figs. 7 and 8, it can be noticed that the deviations of $\langle T \rangle$ and of $f_1$ have the same sign (both positive or negative) at equal values of $\delta m^2$. Such correlation between energy spectrum deviations and time variations of the signal can be traced to the $L/E$ dependence of the vacuum oscillation probability, as recently emphasized in [34].

In conclusion, the SuperKamiokande energy spectrum data constrain rather strongly the vacuum oscillation solutions, allowing only the restricted range $\delta m^2 \simeq 6 - 7.5 \times 10^{-11}$ eV$^2$ (at large mixing). The present experimental sensitivity does not allow detection of semiannual modulations of the neutrino rate due to oscillations. Detection at greater than $1\sigma$ level should require at least one more year of operation, unless the uncertainties get reduced further by improving the energy threshold or the background rate, relative to the present levels.

IV. SUMMARY AND CONCLUSIONS

We have performed a thorough analysis of the solar neutrino problem in two scenarios: MSW and vacuum oscillations (between two neutrino families). We have used the most recent available data, including the preliminary results of the SuperKamiokande experiment after 306.3 days of operation. Particular attention has been devoted to the treatment of the new, solar-model independent data, namely, the energy spectrum information and the time variations of the signal in SuperKamiokande. It has been shown that: 1) the small mixing angle solution is preferred over the large angle one; 2) the vacuum oscillation solutions are strongly constrained by the energy spectrum measurements; and 3) the detection of semiannual modulations of the $^8$B neutrino flux in SuperKamiokande should require at least one more year of operation.

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APPENDIX: EVALUATION OF \langle T \rangle FROM THE SUPERKAMIOKANDE SPECTRUM

In this Appendix we give the detailed derivation of the result reported in Eq. (1). We make use of the preliminary SuperKamiokande spectrum data after 306.3 days of operation \cite{14, 16}, as reported in Table II and shown in Fig. 4.

The average value of the total electron energy \( E_e \) from the unbinned theoretical spectrum \( n(E_e) \) (solid curve in Fig. 4) is given by

\[
\langle E_e \rangle_{\text{theo (unbinned)}} = \frac{\int_{6.5 \text{ MeV}}^{20 \text{ MeV}} dE_e \ n(E_e) \ E_e}{\int_{6.5 \text{ MeV}}^{20 \text{ MeV}} dE_e \ n(E_e)} \approx 8.536 \text{ MeV} .
\] (A1)

Using instead the binned theoretical spectrum \( n_i \) from Table II one obtains:

\[
\langle E_e \rangle_{\text{theo (binned)}} = \frac{\sum_{i=1}^{16} n_i E_i}{\sum_{i=1}^{16} n_i} = 8.525 \text{ MeV} .
\] (A2)

The difference between the values in Eqs. (A1) and (A2) is only \( \approx 0.1\% \), indicating that the effect of binning is not important in the evaluation of the average energy. We will anyway attach to \( \langle E_e \rangle_{\text{theo}} \) a \( \pm 0.010 \text{ MeV} \) “binning error” that, as we shall see, is an order of magnitude smaller than the present experimental uncertainties. Another small error is induced by the \( ^8\text{B} \) neutrino spectrum uncertainty (as shown by the dotted curves in Fig. 4), that we evaluate as \( \Delta \langle E_e \rangle_{\text{theo}} = \pm 0.016 \text{ MeV} \) at the 1\( \sigma \) level.

The average value of \( E_e \) derived from the (binned) experimental spectrum is given by

\[
\langle E_e \rangle_{\text{exp}} = \frac{\sum_{i=1}^{16} n_i r_i E_i}{\sum_{i=1}^{16} n_i r_i} = 8.604 \text{ MeV} ,
\] (A3)

where \( r_i \) and \( E_i \) are given in Table II. In general, the squared error \( \sigma^2_{\langle E \rangle} \) associated to \( \langle E_e \rangle_{\text{exp}} \) reads

\[
\sigma^2_{\langle E \rangle} = \sum_{i=1}^{16} \sum_{j=1}^{16} \frac{\partial \langle E_e \rangle_{\text{exp}}}{\partial r_i} \frac{\partial \langle E_e \rangle_{\text{exp}}}{\partial r_j} \rho_{ij} \sigma_i \sigma_j ,
\] (A4)

where \( \sigma_i \) represents the error of \( r_i \) and \( \rho_{ij} \) is the correlation matrix. The partial derivatives are easily obtained from Eq. (A3):

\[
\frac{\partial \langle E_e \rangle_{\text{exp}}}{\partial r_i} = n_i \frac{E_i - \langle E \rangle_{\text{exp}}}{\sum_{j=1}^{16} n_j r_j} .
\] (A5)

The correlation coefficients for the statistical and uncorrelated systematic errors are given by \( \rho_{ij} = \delta_{ij} \), so that from Eq. (A4) one obtains

\[
\sigma_{\langle E \rangle_{\text{stat. + uncorr. syst.}}} = \pm \left[ \sum_{i=1}^{16} \left( \frac{\partial \langle E_e \rangle_{\text{exp}}}{\partial r_i} \sigma_i \right)^2 \right]^{\frac{1}{2}} = \left\{ \begin{array}{c} +0.108 \\ -0.055 \end{array} \right. \text{ (MeV)} ,
\] (A6)
where the $\sigma'_i$’s are taken from Table II, upper and lower errors being propagated separately. The remaining systematic errors are fully correlated \cite{4}, i.e., $\rho_{ij} = 1$, so that

$$\sigma_{(E)}(\text{corr. syst.}) = \pm \sum_{i=1}^{16} \frac{\partial \langle E_e \rangle_{\text{exp}}}{\partial r_i} \sigma''_i = \begin{cases} +0.170 \\ -0.050 \end{cases} \text{ (MeV)}, \quad (A7)$$

where the $\sigma''_i$’s are given in the last two columns of Table II.

Taking the difference between Eqs. (A3) and (A2) and including all the uncertainties, one obtains

$$\langle E \rangle_{\text{exp}} - \langle E \rangle_{\text{theo}} = 0.079 \pm 0.108 \pm 0.170 \pm 0.010 \pm 0.016 \text{ (MeV)}, \quad (A8)$$

where the first error is due to statistics and to uncorrelated systematics, the second to correlated systematics, the third to the electron spectrum binning, and the fourth to the $^8$B neutrino spectrum uncertainties. In conclusion, passing from total to kinetic energies, it follows that

$$\frac{\langle T \rangle_{\text{exp}} - \langle T \rangle_{\text{theo}}}{\langle T \rangle_{\text{theo}}} = 0.99^{+2.52}_{-0.96} \% , \quad (A9)$$

having added in quadrature the 1$\sigma$ independent errors in Eq. (A8). This is the final result anticipated in Eq. (1).
TABLE I. Neutrino event rates measured by solar neutrino experiments, and corresponding predictions from the BP95 standard solar model. The quoted errors are at 1σ.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Ref.</th>
<th>Data</th>
<th>Theory [6]</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestake</td>
<td>[17]</td>
<td>2.54 ± 0.16 ± 0.14</td>
<td>9.3$_{-1.4}^{+1.2}$</td>
<td>SNU</td>
</tr>
<tr>
<td>Kamiokande</td>
<td>[8]</td>
<td>2.80 ± 0.19 ± 0.23</td>
<td>6.62$_{-1.12}^{+0.93}$</td>
<td>$10^6$ cm$^{-2}$ s$^{-1}$</td>
</tr>
<tr>
<td>SAGE</td>
<td>[18]</td>
<td>73 ± 8.5±5.2</td>
<td>137±8</td>
<td>SNU</td>
</tr>
<tr>
<td>GALLEX</td>
<td>[19]</td>
<td>76.2 ± 6.5 ± 5</td>
<td>137±8</td>
<td>SNU</td>
</tr>
<tr>
<td>SuperKamiokande</td>
<td>[14]</td>
<td>2.44 ± 0.06±0.25</td>
<td>6.62$_{-1.12}^{+0.93}$</td>
<td>$10^6$ cm$^{-2}$ s$^{-1}$</td>
</tr>
</tbody>
</table>

TABLE II. The electron energy spectrum at SuperKamiokande. First three columns: sequential number, energy range, and average energy for each bin. Fourth column: expected number of events $n_i$ per bin per day per kiloton without oscillations (our calculation). Fifth column: ratio of measured to expected rates in each bin. Sixth and seventh columns: Upper and lower 1σ errors of $r_i$ from statistical and uncorrelated systematic uncertainties. Eighth and ninth columns: Upper and lower 1σ errors of $r_i$ from correlated systematic uncertainties. The numbers in columns 6–9 have been graphically reduced from the plots shown in [14–16].

<table>
<thead>
<tr>
<th>$i$</th>
<th>Range $^a$</th>
<th>$E_i$ $^a$</th>
<th>$n_i$ $^b$</th>
<th>$r_i$</th>
<th>$±σ_i^t$ (stat.+unc.)</th>
<th>$±σ_i^c$ (corr. syst.)</th>
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<tbody>
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<td>1</td>
<td>[6.5, 7]</td>
<td>6.747</td>
<td>0.309</td>
<td>0.346</td>
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<td>-0.032</td>
</tr>
<tr>
<td>2</td>
<td>[7, 7.5]</td>
<td>7.244</td>
<td>0.271</td>
<td>0.341</td>
<td>+0.045</td>
<td>-0.032</td>
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<td>3</td>
<td>[7.5, 8]</td>
<td>7.742</td>
<td>0.232</td>
<td>0.350</td>
<td>+0.043</td>
<td>-0.036</td>
</tr>
<tr>
<td>4</td>
<td>[8, 8.5]</td>
<td>8.239</td>
<td>0.198</td>
<td>0.400</td>
<td>+0.062</td>
<td>-0.037</td>
</tr>
<tr>
<td>5</td>
<td>[8.5, 9]</td>
<td>8.742</td>
<td>0.165</td>
<td>0.327</td>
<td>+0.064</td>
<td>-0.039</td>
</tr>
<tr>
<td>6</td>
<td>[9, 9.5]</td>
<td>9.240</td>
<td>0.135</td>
<td>0.373</td>
<td>+0.075</td>
<td>-0.046</td>
</tr>
<tr>
<td>7</td>
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<td>9.736</td>
<td>0.109</td>
<td>0.382</td>
<td>+0.092</td>
<td>-0.049</td>
</tr>
<tr>
<td>8</td>
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<td>0.373</td>
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<td>-0.059</td>
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<tr>
<td>10</td>
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<td>0.050</td>
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<td>-0.062</td>
</tr>
<tr>
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<td>0.036</td>
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<td>-0.072</td>
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<tr>
<td>12</td>
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<td>0.026</td>
<td>0.402</td>
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</tr>
<tr>
<td>13</td>
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<td>0.017</td>
<td>0.359</td>
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<tr>
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<tr>
<td>15</td>
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<td>-0.183</td>
</tr>
<tr>
<td>16</td>
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<td>0.010</td>
<td>0.493</td>
<td>+0.510</td>
<td>-0.170</td>
</tr>
</tbody>
</table>

$^a$Units: MeV (total electron energy).

$^b$Units: events/day/kton/bin.
TABLE III. Number of signal events $N_S$ and signal-to-background ratio $N_S/N_B$ for various \( \cos \theta_{\text{sun}} \) thresholds (as derived from the \( \cos \theta_{\text{sun}} \) event spectrum presented in [14–16]), together with the corresponding uncertainty \( \sigma_f \) of the deviation of the first Fourier coefficient \( f_1 \) from its standard value \( \varepsilon \) [see Eq. (3)].

<table>
<thead>
<tr>
<th>( \cos \theta_{\text{sun}} )</th>
<th>( N_S )</th>
<th>( N_S/N_B )</th>
<th>( \sigma_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 0.5</td>
<td>$\sim$ 4100</td>
<td>$\sim$ 0.30</td>
<td>0.023</td>
</tr>
<tr>
<td>&gt; 0.6</td>
<td>$\sim$ 3870</td>
<td>$\sim$ 0.35</td>
<td>0.022</td>
</tr>
<tr>
<td>&gt; 0.7</td>
<td>$\sim$ 3620</td>
<td>$\sim$ 0.38</td>
<td>0.021</td>
</tr>
<tr>
<td>&gt; 0.8</td>
<td>$\sim$ 3270</td>
<td>$\sim$ 0.59</td>
<td>0.020</td>
</tr>
<tr>
<td>&gt; 0.9</td>
<td>$\sim$ 2530</td>
<td>$\sim$ 0.92</td>
<td>0.020</td>
</tr>
</tbody>
</table>
REFERENCES


[18] SAGE Collaboration, as reported in [14].


FIGURES

FIG. 1. The current state of the solar neutrino deficit. The ellipses represent the regions allowed at 99% C.L. by the present solar neutrino data (dashed lines) and by the standard solar model (SSM) of Bahcall-Pinsonneault [3]. The coordinates are the chlorine (Cl), gallium (Ga), and water-Cherenkov [normalized (K+SuperK)/SSM] neutrino rates.

FIG. 2. The MSW small and large mixing angle solutions to the solar neutrino problem, as obtained by a fit to the total rates only (energy spectrum and night-day data not included).

FIG. 3. The vacuum oscillation solutions to the solar neutrino problem, as obtained by a fit to the total rates only (energy spectrum and night-day data not included). The range of the (logarithmic) vertical scale is $(0.5-2) \times 10^{-10} \text{eV}^2$.

FIG. 4. The electron recoil energy spectrum at SuperKamiokande. The dots represent the measured rates in each energy bin, with vertical 1$\sigma$ error bars given by the quadratic sum of statistical and systematic errors. The horizontal bars span the bin widths. Also shown is the standard spectrum expected in the absence of oscillations (solid line), together with the spectra obtained by assuming $\pm 3\sigma$ deviations of the standard $^8\text{B}$ neutrino spectrum [27]. The data are taken from [14–16]. The theoretical spectra refer to our calculations, renormalized to give the same area as the experimental histogram.

FIG. 5. Constraints on the MSW solutions as derived by the measured value of the average electron kinetic energy $\langle T \rangle$. Upper panel: Theoretical expectations for the fractional shift of $\langle T \rangle$ from its standard (no oscillation) value $\langle T \rangle_0$. Lower panel: regions excluded at 2, 3, and 4 standard deviations by the SuperKamiokande determination of $\langle T \rangle$.

FIG. 6. Constraints on the MSW solutions as derived by the measured value of the asymmetry of nighttime ($R_N$) and daytime ($R_D$) event rates. Upper panel: theoretical expectations for $(R_N - R_D)/(R_N + R_D)$. Lower panel: Regions excluded at 2, 3, and 4 standard deviations by the SuperKamiokande data.

FIG. 7. Constraints on the vacuum oscillation solutions as derived by the measured value of the average electron kinetic energy $\langle T \rangle$. Upper panel: Theoretical expectations for the fractional shift of $\langle T \rangle$ from its standard (no oscillation) value $\langle T \rangle_0$. Lower panel: Regions excluded at 2, 3, and 4 standard deviations by the SuperKamiokande determination of $\langle T \rangle$.

FIG. 8. Fourier analysis of time variations induced by vacuum oscillations. Solid curve: expected deviations of the first Fourier coefficient $f_1$ from its standard value (equal to Earth orbit eccentricity $\varepsilon$) for $\sin^2 2\theta = 1$. Error bars: Our estimate of the statistical errors of SuperKamiokande (including background) after 1, 2, and 4 years of data taking. See the text and [33] for details.
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