Perturbative universality in QCD jet physics

Valery A. Khoze$^{1,2}$

$^{1}$INFN - Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy
$^{2}$Department of Physics, University of Durham, DH1 3LE, UK

Abstract

I survey some recent advances in the applications of the analytical perturbative approach to the description of particle distributions in multi-jet processes. New tests of the perturbatively based picture in the (semi) soft region are discussed.

*Talk at the XXVII International Symposium on Multiparticle Dynamics, Frascati (Rome), Italy, 8-12 September 1997.
1. Introduction

These days the field of multiparticle production in QCD jets has entered a Renaissance age. It looks quite timely to try to realise where we are now and where we are going.

In this talk I focus on some selected aspects of chromodynamics of jets in the (semi) soft region. The main goal is to illustrate some recent impressive phenomenological advances of the analytical perturbative approach (for reviews see e.g. [1, 2, 3]) which attempts to describe the gross features of the hadronic jet-like final states without making any reference to the fragmentation dynamics at all. This approach is based on the so-called Modified Leading Logarithmic Approximation (MLLA) [4] and on the concept of Local Parton Hadron Duality (LPHD) [5].

In the last years physics of hadroproduction in multi-jet events has been very intensively studied in $e^+e^-$, hadron-hadron and $ep$ scattering processes. It will certainly remain one of the main topics for studies at the $e^+e^-$, $pp$ and $ep$ colliders of the future. The interest in the detailed studies of the jet chromodynamics is twofold. On the one hand, they are important for testing both perturbative and non-perturbative physics of multiple hadroproduction, for design of experiments and the analysis of the data. On the other hand, the detailed knowledge of the characteristic features of the multi-jet states could provide useful additional tools to study other physics. For instance, it could play a valuable role in digging out the signals for new physics from the conventional QCD backgrounds using the colour event portrait as a “partonometer” mapping the basic interaction short-distance process (for recent detailed studies and references see [3]).

Nowadays, a vast amount of data from hadronic $Z^0$ decays ($\sim 20$ million events, about 40 identified mesons and baryons) has been accumulated in $e^+e^-$ collisions (for reviews see e.g. [3, 4, 5]). New results continue to pour out from LEP, see [9]. Recently new (very impressive) experimental data on particle distributions in multi-jet events from HERA and TEVATRON have become available, see e.g. [10, 11, 12, 13].

The wealth of existing data collected in various hard processes (in hardness interval $10^1 - 10^5$ GeV$^2$) convincingly proves the dominant role of the perturbative phase of jet evolution and strongly supports the LPHD hypothesis according to which the conversion of partons into hadrons occurs at low virtuality scale (of order of a hadron mass), independent of the scale of the primary hard process, and involves only low-momentum transfer.

The LPHD allows one to relate the (sufficiently) inclusive hadronic observables to the corresponding quantities computed for the cascading partonic system. Only two parameters are actually involved in the perturbative description: the effective QCD scale $\Lambda$ and a cut-off parameter $Q_0$. The non-perturbative effects are practically reduced to normalization constants relating hadronic characteristics to partonic ones. Up to now there were no special reasons to update the values of the free phenomenological parameters found from the first perturbative analysis of the inclusive particle spectra in jets [3].
Rediscovery of coherence in the context of QCD in the early eighties has led to a dramatic revision of theoretical expectations for semisoft particle distributions. Thus, the coherent effects in the intrajet partonic cascades, resulting on the average, in the angular ordering (AO) of sequential branching, gave rise to the *hump-backed* shape of particle spectra — one of the most striking perturbative predictions [3, 14, 15]. It is not the softest particles, but those with the intermediate energies ($E_h \sim E^{0.3-0.4}$) which multiply most effectively in QCD cascades. Due to the interjet coherence which is responsible for the string [16]/ drag [17] effect in the multi-jet hadronic events, a very important physical phenomenon can be experimentally verified, namely, the fact that it is the dynamics of the colour which governs the production of hadrons in accordance with the QCD “radiophysics” of particle flows. Recently the first (quite impressive) data on interjet coherence effects in $W +$ jet production from D0 [18] have become available.

The experimental studies of the structure of the multi-jet events nicely demonstrate that the bright colour interference effects survive the hadronization stage and are clearly seen in the data. This could be taken as a strong argument in favour of the LPHD concept. However, despite all its phenomenological successes, the LPHD is, by no means, a complete theoretical scheme but rather the simplest model-independent approach. Without doubt, the hadronization effects could and should be of importance in many cases. After all, we observe jets of hadrons in the detectors, not the quarks and gluons we are dealing with in our perturbative calculations. However, the dynamics of hadronization is still not well understood from first principles and one has to rely on the predictions of the phenomenological models, which are far from perfect, see e.g. [8, 19, 20]. Moreover, for many inclusive observables the LPHD concept (at least, in its milder formulation) is quantitatively realised within these algorithmic schemes.

It has to be emphasised that the LPHD lies at the very heart of the perturbative approach, but at the same time this key hypothesis could be considered as its Achilles heel. One may expect that LPHD works better and better with increasing energy since the sensitivity to the cut-off should decrease. It seems to be quite a delicate question of where exactly to draw the line of what precisely perturbative picture is capable to predict at current energies and what not. To find out such lines is a challenge to experiment. For instance, one may be tempted to ask an instructive question of what is the largest value of the cut-off $Q_0$ which is allowed by the whole wealth of the present data (inclusive particle spectra and correlations, multiplicity distributions, distributions of event-shape variables, string/drag effect etc). Certainly, the $Q_0$ scale definition depends on the adopted hadronization model. Thus, for instance, within the Lund string scheme [16], the $Q_0$ scale above 2GeV is disfavoured by the existing data [21].

I would expect that the allowable $Q_0$ value could be pushed down towards the hadronic mass scale if one performs the detailed analysis of the data on the dependence of the string/drag effect in $q\bar{q}g$ events on the particle mass $m_h$ and $p_h^\text{out}$ (momentum out of the event plane). In my view this may be an interesting exercise for the QCD fitting experts.
2. On inclusive particle spectra in QCD jets

One of the well known (but still quite impressive) predictions of the perturbative scenario is the *hump-backed* shape of the inclusive particle distributions in the variable $\xi = \log \frac{1}{x}$ with $x = \frac{2E_h}{\sqrt{s}}$. At the moment all observed inclusive energy spectra prove to be in surprisingly good agreement with the predicted by the MLLA-LPHD approximately Gaussian-shape distribution. Moreover, the data collected in various hard scattering processes ($e^+e^-$, DIS, $pp$) clearly demonstrate a remarkable universality of particle spectra assuming the proper (MLLA-based) choice of the cascading evolution variables, equivalent to the $e^+e^- cms$ energy $\sqrt{s}$. Recall that within the QCD cascading picture the evolution parameter corresponding to the struck quark jet in DIS in Breit frame is the four-momentum transfer $\sqrt{Q^2}$ (see e.g. [22, 23]). The proper energy scale for inclusive particle distribution in jets within restricted cone $\theta_0$ measured by the CDF [13] is $E$-jet $\theta_0$, see refs. [2, 24].

The experimental analysis of the current jet hemisphere in DIS is the Breit frame [11] shows that the charged hadron spectrum not only has the same shape as that seen in a single hemisphere of an $e^+e^-$ event but also that this shape evolves in $Q^2$ in the same way as the latter does in terms of the $e^+e^-$ centre-of-mass energy $\sqrt{s}$. The measured area, peak position and the width $\sigma$ of the spectrum confirm that the evolution variable, equivalent to the $e^+e^- cms$ energy $\sqrt{s}$, is in the Breit frame $Q$. The variation of the peak position $\xi_p^*$ with $Q$ follows the $e^+e^-$ curve very closely.

A striking confirmation of the perturbative picture has been found by the CDF [13]. The studies were performed of the inclusive charged particle momentum distributions for a variety of dijet masses ($83 < M_{jj} < 625$GeV) and opening angles $\theta_0$. The shapes of the measured $\xi$-distributions at various $E$-jet $\theta_0$ values turn out to be remarkably close to the MLLA expectations. As $E$-jet $\theta_0$ increases, the peak of the spectrum, $\xi^*$ shifts towards larger values of $\xi$ in perfect agreement with the MLLA predictions and $e^+e^-$ data.

Quite challenging looks the low momentum wing of the particle spectra ($p_h \lesssim 1$GeV) where the non-perturbative dynamics could wash out the perturbatively based expectations. An attempt to stretch the perturbative predictions to the limit of their applicability (or better to say, beyond it) has been performed in [25, 26]. In particular, it was shown that (after the proper modifications) the perturbatively based formulae allow a sufficiently smooth transition into the soft momentum domain. These modifications are closely related to the colour coherence in the parton branchings and to the space-time picture of hadroproduction in QCD jets. Let us recall that the gluons of long wave length are emitted by the total colour current which is independent of the internal structure of the jet and is conserved when the partons split. Applying the LPHD hypothesis one then expects that the hadron spectrum at very low momenta $p$ should be nearly independent of the jet energy [3, 24].

As discussed in [23], the low momentum data could be considered as a further confirmation of the basic ideas of QCD coherence and LPHD. Quantitatively, the analysis was performed in
terms of the invariant particle density \( E \frac{d\eta}{dy} \) for \( e^+e^- \) annihilation into hadrons at low momenta in quite a wide cms energy region (from ADONE to LEP-2). The spectra were found to be in a good agreement with the scaling behaviour and with analytical results. Furthermore, the new H1 data [11] are in a good agreement with the perturbative expectations, thus confirming the universality of soft particle production [26].

We briefly discuss here some selected issues on the inclusive one-particle distributions in jets which were the starting point for the first quantitative tests of the MLLA predictions, see e.g. [2, 3, 5].

Recall that within the MLLA the parton energy spectrum appears as a solution of the corresponding Evolution Equation [2, 4]. This solution can be presented analytically in terms of confluent hypergeometric functions depending on two parameters, the effective QCD scale \( \Lambda \) and the \( k_L \) cut-off \( Q_0 \) in the partonic cascades. When \( Q_0 = \Lambda \) the analytical result simplifies drastically and one arrives at the so-called limiting spectrum [3] which proves to be so successful in fitting the data on charged particle and pion production in QCD jets.

For the case of \( e^+e^- \) collisions the inclusive hadron spectrum is the sum of two \( q \)-jet distributions. In terms of the limiting spectrum one obtains

\[
\frac{1}{\sigma} \frac{d\sigma^h}{d\xi} = 2K^h D_{q}^{\text{lim}}(\xi, Y) \tag{1}
\]

where \( K^h \) is the hadronization constant, \( \sqrt{s} \) the total cms energy and \( Y = \log(\sqrt{s}/2Q_0) \). The limiting spectrum is readily given using an integral representation for the confluent hypergeometric function [4, 27]

\[
D_{q}^{\text{lim}}(\xi, Y) = \frac{4C_F}{b} \Gamma(B) \times \int_{-\pi}^{\pi} \frac{d\ell}{\pi} e^{-B\alpha} \left[ \frac{\cosh \alpha + (1 - 2\zeta)\sinh \alpha}{2N_C^b} Y \frac{\alpha}{\sinh \alpha} \right]^{B/2} \times I_B \left( \sqrt{16N_C^b} \frac{Y}{b} \frac{\alpha}{\sinh \alpha} \left[ \cosh \alpha + (1 - 2\zeta)\sinh \alpha \right] \right).
\]

Here \( \alpha = \alpha_0 + i\ell \) and \( \alpha_0 \) is determined by \( \tanh \alpha_0 = 2\zeta - 1 \) with \( \zeta = 1 - \frac{\xi}{\sqrt{4}} \). \( I_B \) is the modified Bessel function of order \( B \), where \( B = a/b, \; a = 11N_C/3 + 2n_f/3N_C^2, \; b = (11N_C - 2n_f)/3 \), with \( n_f \) the number of flavours and \( C_F = (N_C^2 - 1)/2N_C = 4/3 \).

The analysis of charged particle spectra using this distribution (e.g. [3]) yields values for the effective scale parameter \( \Lambda \equiv \Lambda_{ch} \approx 250 \text{MeV} \). If both parameters \( Q_0 \) and \( \Lambda \) are kept free in the fit one recovers the limiting spectrum with \( Q_0 = \Lambda \) as best solution.

It proves to be very convenient (see e.g. [3, 27]) to analyse inclusive particle spectra in terms of the normalised moments

\[
\xi_q \equiv \langle \xi^q \rangle = \frac{1}{\bar{N}_E} \int d\xi \xi^q D(\xi) \tag{3}
\]

where \( \bar{N}_E \) is the multiplicity in the jet, the integral of the spectrum. These moments characterize the shape of the distribution and are independent of normalisation uncertainties. The
theoretical predictions for the moments from the limiting spectrum are determined by only one free parameter \( \Lambda_{ch} \). Also one defines the cumulant moments \( K_q(Y, \lambda) \) or the reduced cumulants \( k_q \equiv K_q / \sigma^q \), which are related by

\[
\begin{align*}
K_1 & \equiv \bar{\xi} \equiv \xi_1 \\
K_2 & \equiv \sigma^2 = \langle (\xi - \bar{\xi})^2 \rangle, \\
K_3 & \equiv 3 \sigma^3 = \langle (\xi - \bar{\xi})^3 \rangle, \\
K_4 & \equiv k \sigma^4 = \langle (\xi - \bar{\xi})^4 \rangle - 3 \sigma^4
\end{align*}
\]

where the third and fourth reduced cumulant moments are the skewness \( s \) and the kurtosis \( k \) of the distribution. If the higher-order cumulants \( (q > 2) \) are sufficiently small, one can reconstruct the \( \xi \)-distribution from the distorted Gaussian formula, see \([27, 28]\).

The cumulant moments can be obtained from

\[
K_q(Y, \lambda) = \int_0^Y dy \left( -\frac{\partial}{\partial \omega} \right)^q \gamma_\omega(\alpha_S(y)) \mid_{\omega=0}
\]

where \( \gamma_\omega(\alpha_S(y)) \) denotes the anomalous dimension which governs the energy evolution of the Laplace transform \( D_\omega(Y) \) of the \( \xi \)-distribution \( D(\xi, Y) \). In ref. \([27]\) the technique was developed which allows one to derive the analytical expressions for \( \frac{\langle \xi^q \rangle}{Y^q} \).

It is worthwhile to emphasize that the basic MLLA formulae have been derived formally in a high energy approximation. However, even at moderate energies \( \sqrt{s} \) they are expected to give reasonable quantitative predictions because they correspond to the exact solution of the MLLA Evolution Equation which accounts for the main physical ingredients of parton multiplication, namely, colour coherence and energy balance in 2-particle QCD branching, and takes into account also the boundary conditions for low virtuality \( E\theta \). And this expectation proves to be well established experimentally.

As a consequence of colour coherence soft parton multiplication is suppressed, and the \( \xi \)-distribution has the form of a hump-backed plateau which is asymptotically Gaussian in the variable \( \xi \) around the maximum. As was mentioned before, the hump-backed plateau is among the fundamental predictions of perturbative QCD. Its experimental observation was welcomed by the QCD community but without special excitement. Nobody nowadays expects miracles from the results of perturbative calculations. However, better salesmen might be tempted to claim that the spectacular experimental confirmation of the hump-backed plateau in particle spectra has already clearly revealed the drastic low \( x \)-driven violation of the traditional DGLAP expectation \([27]\), a phenomenon which many people in the other experimental environments (e.g. structure functions in DIS) are still so desperately aiming for, see e.g. \([10]\).

A characteristic property of the limiting spectrum \([3]\) is that it approaches a universal finite limit at the phase space boundary \( \xi = \bar{\xi} \) \([30]\).

\[
D_q^{\lim}(Y, Y) = C_q^g D_q^{\lim}(Y, Y) = C_q^g L,
\]
with
\[ C^q_g = \frac{C_F}{N_C} = \frac{4}{9}, \tag{7} \]
\[ L = \frac{4N_C}{a} = 1.069(1.055) \text{ for } n_f = 3(5). \tag{8} \]

An easily accessible characteristic of the \( \xi \)-distribution is its maximum \( \xi^* \) which has been extensively studied by the experimental groups. The high-energy behaviour of this quantity for the limiting spectrum is predicted \[27, 28\] as
\[ \xi^* = Y \left[ \frac{1}{2} + \sqrt{\frac{C}{Y} - \frac{C}{Y}} \right] \tag{9} \]
with the constant term given by
\[ C = \frac{a^2}{16N_Cb} = 0.2915(0.3513) \text{ for } n_f = 3(5). \]

Alternatively, one can compute the maximum \( \xi^* \) from the Distorted Gaussian approximation:
\[ \xi^* = \xi - \frac{1}{2}s\sigma \tag{10} \]

It appears that in the available energy range the expression (9) leads to a nearly linear dependence of \( \xi^* \) on \( Y \). It is worthwhile to mention that in the large \( N_C \) limit, when \( 11N_C \gg 2n_f \) the parameter \( C \) becomes independent on both \( n_f \) and \( N_C \) and approaches its asymptotical value of \( C = \frac{11}{3}\frac{1}{2} \simeq 0.23 \). Therefore in this limit the effective gradient of the straight line is determined by such a fundamental parameter of QCD as the celebrated \( \frac{11}{3} \) factor (characterizing the gluon self interaction) in the coefficient \( b \).

As has already been mentioned, formula (9) describes surprisingly well the observed evolution of the maximum of the spectra measured in \( e^+e^- \) collisions, current jet fragmentation at HERA and in the dijet events at TEVATRON (assuming a proper choice of the cascading variable). The existing experimental results on the \( \xi^* \) evolution prove to be completely inconsistent with cylindrical phase space expectations, see e.g. \[3, 10, 11\].

Let us make here a few comments concerning the application of the perturbative analytical results to the identified particle distributions. Recall that in the context of the LPHD logic the limiting formulae are applied for dealing with the inclusive distributions of the “massless” hadrons (\( \pi \’s \)) and for all charged particle spectra. To approximate the distributions of “massive” hadrons (\( K, \rho, p \ldots \)) the partonic formulae truncated at different cut-off values \( Q_0(Q_0(m_h) > \Lambda) \) could be used, e.g. \[2, 3\]. Within the framework of the LPHD-MLLA picture there is no recipe for relating \( Q_0 \) to the masses of the produced hadrons and their quantum numbers. One needs further detailed phenomenological studies of the \( Q_0 \) dependence of the spectra of identified particles/resonances. Here also the data on different hadron species from jets at the TEVATRON and HERA would be very helpful.
The analytical expressions for the truncated parton distributions representing the exact solution of the Evolution Equation [2, 4, 3] are not transparent for physical interpretation and are not easily suited to straightforward numerical calculations. However, one can represent the results in terms of distorted Gaussian distribution for \( D(\xi, Y, \lambda) \) with \( \lambda = \log \frac{Q_0}{\Lambda} \). The MLLA effects are encoded in terms of the analytically calculated (for \( Q_0 \neq \Lambda \)) shape parameters \( \xi, \sigma, k, s \), see eq. (4). The mean parton multiplicity can be written [5] in a compact form in terms of modified Bessel (MacDonald) functions \( I_\nu(x) \) and \( K_\nu(x) \),

\[
N_A(Y, \Lambda) = C_g^g x_1 \left( \frac{z_2}{z_1} \right)^B [I_{B+1}(z_1)K_B(z_2) + K_{B+1}(z_1)I_B(z_2)],
\]

(11)

\[
z_1 = \sqrt{\frac{16N_C}{b} (Y + \lambda)}, \quad z_2 = \sqrt{\frac{16N_C}{b} \lambda}
\]

(12)

Here \( A = q, g \) denotes the type of jet (\( C_g^g = 1, C_g^q = \frac{C_F}{N_C} \)). The first term in square brackets in (11) increases exponentially with \( \sqrt{Y} \) while the second term decreases. Its role is to preserve the initial condition for the jet evolution, namely, \( N_g = 1 \).

MLLA predicts the energy independent shift of the peak position for truncated parton distributions as compared to the limiting spectrum [27]. The present data on the identified particle spectra well confirm the perturbative expectation that for different particle species the energy dependence of \( \xi^* \) is universal.

The MLLA-LPHD predictions have been successfully confronted with the data on the identified particle distributions (see e.g. [3, 7, 8] and references therein). In particular, the bell-shaped form of the spectra and their energy evolution are in a fairly good agreement with the perturbative predictions.

Finally, let us turn to the tests of the perturbatively based picture in the soft region, see [25, 26, 27]. Without doubt, it is not a priori clear at all, whether one can appeal to the perturbative expertise when exploring the low momentum domain. However, an attempt to stretch the perturbative expectations to the limit of their applicability looks quite intriguing. This could, in principle, provide a clue for understanding of some conceptual problems of the LPHD. Whether or not the transition between two stages of jet development is soft is a question for experiment.

Certainly, within the perturbative framework there is no unique recipe of how to modify the relation between parton and hadron distributions in order to enter smoothly the soft domain, see discussion in [25, 26]. Here we shall follow an ancient route proposed in [31] (see also [26, 27]) which is based on the phase-space arguments.

Let us recall that at low momenta the invariant density of hadrons \( E^d d\sigma_p \) can be rewritten as

\[
E \frac{dn}{d^3p} \sim \frac{W_1(s, E\sqrt{s})}{s},
\]

(13)
where \( W_i(s, E\sqrt{s}) \) are the standard \( e^+e^- \) analogues of the DIS structure functions \( W_i(q^2, \nu) \). As well known, \( \overline{W}_i(s, E\sqrt{s}) \) are related to the matrix elements of the current commutators and should be regular when \( p \to 0 \). It is then a general requirement that the hadronic density \( E\frac{dn}{d^3p} \) approaches a constant limit when \( p \to 0 \). As demonstrated in Refs. \[25, 26\] this is well established experimentally.

In Ref. \[27\] a simple prescription has been discussed of how to modify (1) in order to satisfy (13) at low particle momenta.

\[
\frac{1}{\sigma} \frac{d\sigma^h}{d \log p} = 2K^h \left( \frac{p}{E} \right)^3 D^\text{lim}_{q}(\xi_E, Y),
\]

with \( \xi_E = \log \frac{\sqrt{s}}{2E} \).

With this prescription one arrives at the following expression for the invariant hadronic density in the case of \( e^+e^- \) annihilation

\[
E\frac{dn}{d^3p} = 2K^h \left( \frac{1}{4\pi E^2} \right) D^\text{lim}_{q}(\xi_E, Y)
\]

As it is easy to see from eqs. (6) and (15), hadronic density approaches a constant limit at \( p \to 0 \).

For large energies \( E \gg \Lambda \) Eqs. (14) and (15) coincide with the standard MLLA-LPHD relations. Let us recall that the low momentum region in charged particle spectra is dominated by pions and that the MLLA limiting spectrum provides a fairly good description of pion spectra at relativistic energies with

\[
K^\pi \simeq 1.1 \text{ and } Q_0 = \Lambda \simeq 150\text{MeV},
\]

see e.g. \[3, 27\].

It is interesting to note that with these parameters the invariant pion density at the very edge of the phase space is given by

\[
E^\pi\frac{dn}{d^3p} = \frac{8}{9} K^\pi 4N_C \frac{1}{a} \frac{1}{4\pi Q_0^2} \approx
\approx 0.9 \frac{1}{4\pi m^2_\pi} \approx 4\text{GeV}^{-2}
\]

It looks quite challenging that the perturbatively based result (17) appears to be so close to the natural hadronic scale for the pion density. Moreover, as shown in Refs. \[23, 26\], the experimental data on soft pion (and soft charged particle) production seem to favour the limiting value (17).

As has already been mentioned the observed production rates of soft particles have proven to be practically independent of the energy of parent parton. Such scaling behaviour has been nicely demonstrated in both \( e^+e^- \) and DIS interactions, over a wide range of \( cms \) energies,
in which the data move toward a common limit as the particle momenta become small, see [11, 23, 26]. This could be considered as strong evidence in support of the LPHD. Recall that the LPHD is deeply rooted in the space-time picture of the hadroproduction in the QCD cascades, e.g. [4]. Thus, within it, in the process \( e^+e^- \rightarrow q\overline{q} \) the first hadrons are formed at the time \( t \sim t_{\text{crit}} \sim R \) (\( R \approx 1 \text{ fm} \) is a characteristic space-time scale of the strong interactions) with \( p \sim p_\perp \sim R^{-1} \sim \Lambda \). It is the moment when the distance between the outgoing \( q \) and \( \overline{q} \) approaches \( R \). At \( t > t_{\text{crit}} \) the two jets are separated as globally branched, and the parton cascades develop inside each of them. The gluon bremsstrahlung becomes intensive only when the transverse distance between any two colour partons exceeds \( R \).

With increasing time the partons with larger and larger energies \( E \sim \frac{t}{R^2} \) hadronize (inside-outside chain). In this picture soft particles with \( E \sim R^{-1} \) produced at the lower edge of the perturbative phase space play a very special role. Their production rate is practically unaffected by the QCD cascading, and their formation is a signal of switching on the real strong interactions (\( \alpha_S \sim (1) \)). In some sense these particles can be considered as the eye-witnesses of the beginning of the “hadronization wave”.

## 3. Colour related phenomena in multi-jet events

It was realised long ago (see e.g. [2] and references therein) that the overall structure of particle distributions in multi-jet events in hard scattering processes (event portrait) is governed by the underlying colour dynamics at short distances. The existing experimental data clearly show in favour of interjet colour coherence, see e.g. [3, 7, 8]. Here we shall briefly discuss some new results on QCD radiophysics of particle flows in multi-jet events, see also [32]. The main lesson from the recent impressive studies is that now we have (quite successfully) entered the stage of quantitative tests of the details of colour drag phenomena.

The interjet coherence phenomena were intensively studied at LEP1, TRISTAN and TEVatron. Let us mention a few new facts concerning comparison with the analytical QCD predictions.

DELPHI [33] has performed the first quantitative verification of the perturbative prediction [17] for the ratio \( R_\gamma \)

\[
R_\gamma = \frac{N_{\gamma\pi}(q\overline{q}g)}{N_{\gamma\pi}(q\overline{q}\gamma)}
\]

of the particle population densities in the interquark valley in the \( e^+e^- \rightarrow q\overline{q}g \) and \( e^+e^- \rightarrow q\overline{q}\gamma \) events. For a clearer quantitative analysis a comparison was performed for the \( Y \)-shaped symmetric events using the double vertex method for the \( q \)-jet tagging. The ratio \( R_\gamma \) of the charged particle flows in the \( q\overline{q} \) angular interval \([35^\circ, 115^\circ]\) was found to be

\[
R_\gamma^{\text{exp}} = 0.58 \pm 0.06.
\]
This value is in a fairly good agreement with the expectation following from [17] at \( N_C = 3 \), for the same angular interval

\[
R^\text{th}_\gamma \approx \frac{0.65N_C^2 - 1}{N_C^2 - 1} \approx 0.61.
\] (20)

The string/drag effect is quantitatively explained by the perturbative prediction and the above ratio does not appear to be affected by hadronization effects in an essential way.

Another new result [33] concerns the analysis of the threefold symmetric \( e^+e^- \rightarrow q\bar{q}g \) events using the double vertex tagging method. It is shown that the string/drag effect is clearly present in these fully symmetric events and it cannot be an artefact due to kinematic selections. Quantitatively, comparing the minima located at \( \pm[50^\circ, 70^\circ] \), the particle population ratio \( R_g = N_{qg}/N_{q\bar{q}} \) in the \( q-g \) and \( q-\bar{q} \) valleys is measured to be

\[
R^{\text{exp}}_g = 2.23 \pm 0.37
\] (21)

This is to be compared with the asymptotic prediction \( R_g = 2.46 \) for projected rates at central angles, whereas for the above angular interval one finds [8, 17]

\[
R^\text{th}_g \approx 2.4.
\]

in good agreement with the experimental value.

If one allows for arbitrary 3-jet kinematic configurations new information can be obtained about the evolution of the event portrait with the variation of event topology, see [3, 34]. Recently ALEPH [35] and DELPHI [36] have demonstrated that, in agreement with the QCD radiophysics [2], the mean event multiplicity in three jet events depends both on the jet energies and on the angles between the jets. These results clearly show the topological dependence of jet properties which was predicted analytically.

Identification of charged hadrons (\( \pi^\pm, K^\pm \) and \( p \)) has allowed ALEPH [37] to study mass dependence of the interjet \( R_g \) values. In full agreement with the perturbative expectations [17, 2, 3] there is no strong mass dependence at LEP-1 energies.

Finally, let us note that L3 and OPAL [38] have studied the dependence of the colour drag on out-of-plane momentum \( p^{\text{out}} \). In agreement with the predictions of [17] the dependence on \( p^{\text{out}} \) was found to be significantly weaker than at lower energies. Recall that the dependence of the magnitude of the string/drag effect on \( p^{\text{out}} \) (and registered particle mass) has to vanish asymptotically in the perturbative approach.

Recently the D0 Collaboration has reported the first results on colour coherence studies in \( W + \text{jet} \) events [18]. One of the instructive measurements concerns the ratio for soft particle production in the event plane to the transverse plane. This quantity proves to be insensitive to the overall normalization of the individual distributions and to detector effects. The experiment shows very good agreement with the perturbative expectation [8] for this ratio.
The clear observation of interjet interference effects gives another strong evidence in favour of the perturbative picture of multihadron production. The collective nature of multi-parton final states reveals itself here via the QCD wave properties of the multiplicity flows. The detailed experimental studies of the colour-related effects are of particular interest for better understanding of the dynamics of hadroproduction in the multi-jet events. For instance, under special conditions some subtle interjet interference effects, breaking the probabilistic picture, may even become dominant, see Refs. [17, 39]. We remind the reader that QCD radiophysics predicts both attractive and repulsive forces between the active partons in the event. Normally the repulsion effects are small, but in the case of colour-suppressed $O(\frac{1}{N_c})$ phenomena they may play a leading role. It should be noted that in Refs. [17, 39] the interjet collective effects were viewed only on a completely inclusive basis, when all the constituents of the multi-element colour antenna are simultaneously active. A challenging possibility to operate within the perturbative scenario with the complete collective picture of an individual event (at least at very high energies) was discussed in [40]. The topologometry on the event-by-event basis could turn out to be more informative than the results of measurements averaged over the events.

Recall, that there is an important difference between the perturbative radiophysics and the parton-shower Monte Carlo models. The latter not only allow but even require a completely exclusive probabilistic description. Normally (such as in the case of $e^+e^- \rightarrow q\bar{q}g$) the two pictures work in a quite peaceful coexistence; the difference only becomes drastic when one deals with the small colour-suppressed effects.

Let us emphasize that the relative smallness of the non-classical effects by no means diminishes their importance. This consequence of QCD radiophysics is a serious warning against the traditional ideas of independently evolving partonic subsystems. So far (despite the persistent pressure from the theorists) no clear evidence has been found experimentally in favour of the non-classical colour-suppressed effects in jets, and the peaceful coexistence between the perturbative interjet coherence and colour-topology-based fragmentation models remains unbroken. However, these days the colour suppressed interference effects attract increased attention. This is partly boosted by the findings that the QCD interference (interconnection) between the $W^+$ and $W^-$ hadronic decays could affect the $W$ mass reconstruction at LEP-2, see e.g. [41].

Finally, let us recall that the colour-related collective phenomena could become a phenomenon of large potential value as a new tool helping to distinguish the new physics signals from the conventional QCD backgrounds (e.g. [2, 3, 6]).

4. Conclusion

During the last few years the experiments have collected exceedingly rich new information on the dynamics of hadronic jets — the footprints of QCD partons. New QCD physics results from LEP2, TEVATRON and HERA continue to pour out shedding light on various aspects of hadroproduction in multi-jet events. The existing data convincingly show that the analytical perturbative approach to QCD jet physics is in a remarkably healthy shape.
The key concept of this approach is that the conversion of partons into hadrons occurs at a low virtuality scale, and that it is physics of QCD branchings which governs the gross features of the jet development. Thus, the perturbative universality of jets appearing in different hard processes is nicely confirmed. Moreover, the data demonstrate that the transition between the perturbative and non-perturbative stages of jet evolution is quite smooth, and that the bright colour coherence phenomena successfully survive the hadronization stage.

LEP1 proves to be a priceless source of information on the QCD dynamics. It has benefited from the record statistics and the substantial lack of background. We have learned much interesting physics, but the need for further detailed analyses of the data recorded at LEP1 has not decreased.

The programs of QCD studies at LEP2 and at future linear $e^+e^-$ colliders look quite promising. The semisoft QCD physics becomes one of the important topics for investigation in the TEVATRON and HERA experiments.

Concluding this talk let me emphasize once more that, of course, there is no mystery within the perturbative QCD framework. One is only supposed to perform the calculational routine properly. So it is entirely unremarkable that the quantum mechanical interference effects should be observed in the perturbative results. Of real importance is that the experiment demonstrates that the transformer between the perturbative and non-perturbative phases acts very smoothly. This message could (some day) shed light on the mechanism of colour confinement.

Acknowledgements


References


[21] The author is very grateful to G. Rudolph for the illuminating discussion concerning fitting of QCD model parameters.


[30] The author is very grateful to Yu. Dokshitzer who has dug the limiting value $L$ out of the pile of old MLLA-related notes.


