Dynamics of Dissipative Binary Collisions

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Abstract

In this paper we discuss the reaction mechanisms that occur in the overlap zone for semi-peripheral heavy ion collisions at intermediate energies. In particular we focus on the development of neck instabilities, that could determine a possible increase of dynamical fluctuations. As observed in recent experimental data, at beam energies just above 10 MeV/A the most relevant expected consequence is the possibility to obtain large variances in the projectile-like and target-like observables. With increasing beam energy we pass to a mid-rapidity fragment production. In this way we predict a smooth transition from a deep-inelastic to a fragmentation reaction mechanism.

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I. INTRODUCTION

In recent years the study of the reaction mechanisms involved in heavy ion collisions at intermediate energies has been the subject of several experimental and theoretical investigations all around the world. In particular, recent experimental results have raised the attention on the possibility to reveal prompt intermediate mass fragment (IMF) production, as well as large fluctuations in the observables related to projectile-like (PLF) and target-like (TLF) fragments in semi-peripheral collisions [1].

In this paper we will concentrate on the dissipation mechanisms occurring in medium momentum-transfer reactions, corresponding to semi-peripheral impact parameters, at beam energies slightly above 10 MeV/A. In this energy range, interesting results on the excitation energy sharing between PLF and TLF fragments and on the observation of large mass variances have been recently reported in Ref. [2].

We will investigate the dynamics of the nuclear overlap zone (the "neck" region): the occurrence of volume instabilities in this zone helps the break-up of the dinuclear system formed in the earlier stage of the reaction, leading to two primary fragments in the exit channel. Volume instabilities can develop due to the coupling between stochastic nucleon-nucleon collisions and the nucleon exchange process already present also at lower energies and extensively studied in deep inelastic collisions and fusion-fission events, from both theoretical and experimental points of view [3–5]. We will discuss the relative importance of these two sources of dissipation and fluctuations in the energy range considered. In particular, the reaction that we will consider here, $^{100}\text{Mo} + ^{120}\text{Sn}$ at 14 MeV/A, lies in the beam energy region where two-body collisions have just started to play a role and, consequently, as we will see in the following, the dissipation mechanism is mostly determined by the one-body nucleon exchange. According to the calculations presented below, at this energy fluctuations due to two-body collisions just start to be important.

The detectable consequence of the occurrence of instabilities should be a clear increase of the variances of all observables (masses, charges, angles, velocities, angular momenta...) of projectile-like (PLF) and target-like (TLF) fragments and, at higher energies, the possibility
of IMF formation from the neck region.

II. DYNAMICAL EVOLUTION

The main purpose of this paper is to study the mechanisms which occur when the di-nuclear system, formed in the earlier stage of the reaction, breaks up into pieces. The system will easily split apart if, after the shock and the initial compression, the rarefaction phase leads the density in the overlap zone below the critical density \( \rho_c \). The occurrence of volume instabilities can therefore explain the break-up of the system in two primary fragments (like in deep-inelastic collisions), or even determine a prompt IMF emission from the neck region at higher bombarding energies, where the nucleon-nucleon collision rate becomes more important \( \mathcal{E} \).

It is well known that, when instabilities are present, fluctuations become extremely important. In fact, in unstable situations, fluctuations are usually amplified and may lead the system towards various patterns that are different from the one associated with the mean trajectory behaviour. In the following we will try to relate the presence of volume instabilities to the possibility, due to the growth of fluctuations, to obtain large variances for the observables associated with projectile-like (PLF) and target-like (TLF) fragments in semi-peripheral reactions at around 15 \( MeV/A \).

A parameter of crucial importance is the time interval during which the di-nuclear system interacts and exchanges nucleons before its break-up. This time essentially depends on impact parameter and beam energy. If it is long compared to the characteristic time for the growing of spinodal instabilities (\( \tau \approx 100 - 200 \; fm/c \) \( \mathcal{F} \)), the effect of the enhancement of fluctuations due to instabilities will be averaged out by the mean-field propagation; in this case we expect to observe just the equilibrium fluctuations associated with the stochastic nature of nucleon exchange and/or nucleon-nucleon collisions in stable systems \( \mathcal{F} \mathcal{F} \). This is for instance the case of deep-inelastic collisions at low energy, below 10 \( MeV/A \). This corresponds mostly to the situation where, in average, PLF and TFL have the same temperature, i.e. the total excitation energy is shared according to the mass ratio of the two spectators.
On the other hand, if the interaction time is of the same order of magnitude of the instability growth time, these fluctuations will be amplified and will lead to variances larger than those expected on the basis of statistical equilibrium, for all observables related to the primary products of the reaction. This is extremely interesting since recent experimental results have already raised the attention on the possibility to obtain a big variety of masses of PLF and TLF, as well as IMF emission from the neck region in semi-peripheral heavy ion collisions at intermediate energies \[1\], where the condition on the interaction time discussed before surely applies.

III. THEORETICAL FRAMEWORK

In order to have a dynamical description that incorporates fluctuations, we will perform calculations considering a stochastic mean field approach. In such a kind of theories the nuclear system is still described by its one body density function in phase space \( f(\mathbf{r}, \mathbf{p}, t) \), while this function may experience a stochastic evolution in response to the action of a fluctuating source term, in some analogy with the Brownian motion.

Recently kinetic one-body equations of the Boltzmann-Nordheim-Vlasov (BNV) (or Boltzmann-Uehling-Uhlenbeck (BUU)) type have been extended by the introduction of a fluctuating term, coming from considerations associated with the random nature of the nucleon-nucleon collision integral \[10–12\]. The resulting Boltzmann-Langevin (BL) equation reads:

\[
\frac{\partial f}{\partial t} + (\mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial U}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{p}}) = \bar{I}[f] + \delta I[f]
\]

(3.1)

where \( U[f] \) is the self-consistent mean field potential, \( \bar{I} \) represents the average effect of the collisions, while \( \delta I \) denotes the fluctuating remainder (the Langevin term).

In our calculations we will use a simplified approach: once local volume instabilities are encountered, we implement in the code local fluctuations of the density according to the amplitude predicted by the BL theory at the temperature and density considered \[10\]. This procedure is described in details in Ref. \[13\]. These fluctuations are then amplified and lead to several situations of mass and excitation energy exchange in the exit channel.
The BNV equation is solved within the test particle method \cite{14}, using the code TWINGO \cite{15}. The following mean-field parameterization has been considered:

\[ U(\rho) = A \left(\frac{\rho}{\rho_0}\right) + B \left(\frac{\rho}{\rho_0}\right)^\sigma, \]

with \( A = -356 \text{ MeV}, \ B = 303 \text{ MeV}, \ \sigma = 7/6 \), that gives a "soft" equation of state, with a compressibility modulus \( K = 200 \text{ MeV} \). \( \rho_0 \) is the equilibrium density of symmetric nuclear matter. We have checked that the numerical fluctuations introduced in this way are negligible when compared to the physical fluctuation amplitude that we implement when instabilities are encountered.

\section*{IV. DESCRIPTION OF RESULTS}

Our aim is mainly to compare the results on the energy sharing between PLF and TLF with the experimental findings obtained recently at GSI \cite{2} for the reaction \(^{100}\text{Mo} + ^{120}\text{Sn}\) at 14 \text{MeV}/A. Because of intrinsic limits of the apparatus, in those experiments observables related to PLF were evaluated in direct and reverse kinematics. We have investigated the evolution of the BNV dynamics from semi-peripheral to peripheral collisions.

At impact parameters less than \( b = 7 \text{ fm} \) the system follows the average path of incomplete fusion, converting the incoming energy partly into rotational motion of the di-nuclear system and partly into internal excitation energy. Increasing the impact parameter the average trajectory of the system evolves towards deep-inelastic process configurations. We observe a binary mechanism, that preserves the identity of the two colliding nuclei, but converting a quite large fraction of the available energy and angular momentum into thermal energy and intrinsic spin of the primary fragments. As mentioned above, we are mainly interested in evaluating the energy sharing between the two partners. Two situations are usually indicated: The quasi-elastic limit and the thermal limit. In the quasi-elastic case the partners come in contact for a relatively short time, allowing only few nucleons to pass from one nucleus to the other. Concerning the excitation energy sharing, this leads in average to the equipartition between the two primary fragments. This result is found in general models, as the Nucleon Exchange Model (NEM) \cite{16}, where, due to the stochastic nature of
the nucleon exchange process, the two partners have fluctuating masses, but essentially the same excitation energy.

On the other side, when the collision time is long enough, the two reaction partners can exchange a very large number of nucleons and the system goes towards thermalization. As a consequence, the excitation energy is divided between the fragments proportionally to their masses. This is what is usually called the thermal limit.

The energy dissipated in the reaction is given essentially by the total kinetic energy loss ($T_{\text{KE}}$), defined as the difference, in the center-of-mass reference frame, between the initial available kinetic energy $E$ and the total kinetic energy ($T_{\text{KE}}$) of the two primary fragments in the exit channel:

$$T_{\text{KE}} = E - T_{\text{KE}}.$$  \hspace{1cm} (4.1)

In the calculation of $T_{\text{KE}}$ we have taken into account the Coulomb repulsion between the two primary fragments in the final stage. The $T_{\text{KE}}$ strongly depends on the impact parameter and the interaction time (the time after which the fragments separate), as we show in Table I.

Because of this, from the experimental point of view, $T_{\text{KE}}$ represents a good parameter to explore the evolution of the system between the two limits of reaction mechanisms indicated above.

**A. Calculations including fluctuations**

In order to perform a comparison with the experimental data [2], we study the correlations that can arise between the net-mass transfer and the excitation energy of PLF and TLF. To tackle this problem we stress that it is necessary to perform an event-by-event analysis, since the fluctuations around the average dynamics play an important role. We have concentrated our analysis on the nature of the binary processes occurring at $b = 8.5, 9, 10$ fm. In Fig.1 we show the time evolution of three events corresponding to these values of $b$. The $T_{\text{KE}}$'s corresponding to these values of the impact parameter are indicated in Table I. It can also be seen in the Table that the separation time is still quite long in the $b = 8$ fm case.
and this complicates significantly the analysis. For this reason we show results starting from $b = 8.5 \text{ fm}$. For each impact parameter several events have been considered. The calculations show that the system in average evolves from the quasi-elastic to the thermal limit along with the rise of $T_{KE}$. Indeed, in the case of the more peripheral collision ($b = 10 \text{ fm}$) the excitation energy is, in average, equally shared between the two fragments, while for $b = 8.5 \text{ fm}$ the largest fragment is more excited. Following the entire dynamics of the system, it is possible to evaluate how many nucleons are exchanged in the three cases. This average number is shown in Table II, along with the $T_{KE}$ bins, as a function of the impact parameter.

In the Table II are shown also the average masses of the fragments and their variances. As expected, the number of exchanges increases rapidly with the dissipation degree of the reaction. The variances observed are only due to dynamical fluctuations, that develop as soon as the di-nuclear system encounters volume or shape instabilities. As mentioned before, this kind of fluctuations is accounted for in our stochastic simulations. However, it should be noticed that, also in stable situations, equilibrium fluctuations are present due to the stochastic nature of the nucleon exchange process. Because of the use of test particles for solving the transport equation, these fluctuations are reduced by a factor $1/N_{\text{test}}$, being $N_{\text{test}}$ the number of test particles per nucleon. We incorporate also these fluctuations (that we will call "statistical" fluctuations), by implementing each stochastic event calculation by the procedure of random clustering of the one-body distribution, introduced in Ref. [17]. The method is widely discussed also in Ref. [4]. Starting from a dynamical event we get many "statistical" events, each of them constructed by randomly choosing a sample of $N_p = A_p - Z_p$ and $Z_p$ test particles among all test particles associated with neutrons and protons of the projectile, and similarly for the protons and neutrons of the target. Then the primary fragments are reconstructed using a coalescence procedure described in Ref. [17]. In this way we reconstruct the "statistical" variances, that are essentially given by the average number of exchanged nucleons (Ref. [4]). As it can be seen from Table II, these fluctuations are larger than the dynamical variances.
B. Comparison with experimental data

In the experimental data (Ref. [2]), it has been unambiguously demonstrated in a model independent way that a correlation between the net mass transfer and the excitation energy of the fragments exists even for relatively high energy dissipation. Moreover in that paper the authors show that, also in events with the two primary fragments having the same mass, the excitation energy is not equally shared between the two fragments, being much more excited the one that gains nucleons. Such a result is in disagreement with both the quasi-elastic limit, usually described by the NEM, and the thermal limit, which predict the equipartition of the excitation energy for equal-mass events. Hence Casini et al. (Ref. [2]) advocate the existence of important dynamical effects in order to explain the data.

As reported in Table II, in our simulations we observe non-negligible fluctuations that come from dynamical effects (neck instabilities), that are responsible for the variances indicated in the Table and many nucleon exchanges between the two reaction partners. Once the statistical fluctuations have been implemented (according to the procedure [17] recalled above), we observe, in each dynamical event, a broadening of the mass distribution according to the formula $\sigma^2_{\text{stat}} = \bar{n}_{\text{exch}}$. One can notice that the ”statistical variances” are larger that the ones given by dynamical instability effects. Indeed, at 14 $MeV/A$, dynamical instabilities have just started to play a role. After introducing the ”statistical” fluctuations, we can now calculate the masses and the excitation energies of the two partners event by event. In Fig.2 we show the excitation energy of the PLF and TLF fragments, obtained at $b = 9$ fm, as a function of their masses. It is possible to see that, as already stressed before, in absence of net mass transfer (i.e. for $A_{PLF} = 100$, $A_{TLF} = 120$), we observe the same excitation energy for the two reaction partners. Then we see that the fragment that receives a given amount of nucleons from the partner is more excited, in agreement with the trend observed in the experimental data. However, it should be noticed that in our calculations, this result does not come from dynamical effects, as suggested in Ref. [2], because we have shown that dynamical variances are small compared to the statistical ones. Hence we conclude that in our calculations this kind of correlations arises from statistical processes, namely from the...
nucleon exchange process.

Actually, as stressed in recent publications [18], this feature can be understood also within the NEM. It is often argued that the NEM predicts equal energy sharing between the two primary fragments (see Ref. [16]). However this is true only if the net mass drift is much smaller than the total diffusion of nucleons. If this condition is not fulfilled a correlation between excitation energy and net mass transfer can arise. This is related essentially to the fact that in a single exchange the hole excitation induced by the leaving nucleon in the donor nucleus is much lower than the particle excitation created in the recipient nucleus. So an asymmetry in the fragment excitation energy is expected in events with a net mass transfer, being more excited the nucleus that gains nucleons. If the total number of exchanges is not much higher than the net transfer, this asymmetry is detectable.

The behaviour observed at $b = 10 \text{ fm}$ is similar to what is obtained at $9 \text{ fm}$ (equipartition of excitation energy between the primary fragments) but variances are much less.

At $b = 8.5 \text{ fm}$ the system goes towards the equal temperature limit; hence the excitation energies of the fragments result to be just proportional to their masses. This trend is also present in the data, when increasing the $TKEL$, even if the equal temperature limit is reached for higher values of $TKEL$.

As one can see from Fig.2, the experimental mass variances are not correctly reproduced in the calculations, since the range of masses detected as PLF and TLF is broader in the data. This could indicate a more important contribution of dynamical effects and instabilities that, in our calculations, at $14 \text{ MeV/A}$ just start to appear.

We expect to find such a contribution when rising the energy. Indeed, calculations at higher energies ($25 \text{ MeV/A}$) are presently in progress and seem to suggest that dynamical instabilities play an important role in the evolution of the system, giving a very relevant contribution to the final variances.
V. CONCLUSIONS

We have investigated the dynamics of semi-peripheral heavy ion collisions at energies around 15 MeV/A. The interest of this kind of study lies on the fact that in recent experimental data large fluctuations have been measured in the observables related to PLF and TLF fragments, that cannot be explained just on the basis of thermal "equilibrium" processes. The dynamical mechanisms and the possible occurrence of instabilities have been investigated, in the case of the reaction $^{100}$Mo + $^{120}$Sn at 14 MeV/A, in the framework of a stochastic mean-field approach. We show that dynamical effects, related to the occurrence of volume and shape instabilities in the neck region, determine an increase of the variances, but at 14 MeV/A this effect is small when compared to the broadening due to statistical fluctuations. At the energy considered dynamical fluctuations just start to play a role. On the basis of previous calculations, larger effects are predicted at higher energy [10], up to cluster formation in the ”neck” region, with variances much above the statistical evaluation. In this way we expect a quite smooth transition in the reaction mechanism for dissipative collisions, from deep-inelastic to fragmentation: ”Natura non facit saltus” [19].

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REFERENCES


TABLE I. The average total kinetic energy loss, calculated in the BNV simulations, as a function of the impact parameter. The average time needed for the separation of the two primary fragments is also reported.

<table>
<thead>
<tr>
<th>$b$ (fm)</th>
<th>$TKEL$ (MeV)</th>
<th>Interaction time (fm/$c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>453</td>
<td>450</td>
</tr>
<tr>
<td>8.5</td>
<td>387</td>
<td>350</td>
</tr>
<tr>
<td>9</td>
<td>347</td>
<td>300</td>
</tr>
<tr>
<td>9.5</td>
<td>272</td>
<td>240</td>
</tr>
<tr>
<td>10</td>
<td>194</td>
<td>210</td>
</tr>
<tr>
<td>10.5</td>
<td>138</td>
<td>180</td>
</tr>
<tr>
<td>11</td>
<td>93</td>
<td>150</td>
</tr>
</tbody>
</table>

TABLE II. The TKEL bin, the statistical mass variance, the average PLF mass and associated dynamical variance, the average TLF mass and associated dynamical variance, calculated in the stochastic simulations, as a function of the impact parameter. The statistical variance is given by the average number of exchanged nucleons (see text).

<table>
<thead>
<tr>
<th>$b$ (fm)</th>
<th>$TKEL$ bin (MeV)</th>
<th>$\sigma^2_{stat}$</th>
<th>$A_{PLF}$</th>
<th>$\sigma^2_{PLF}$</th>
<th>$A_{TLF}$</th>
<th>$\sigma^2_{TLF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>369.4–404.3</td>
<td>63.7</td>
<td>99.1</td>
<td>2.69</td>
<td>120.7</td>
<td>2.69</td>
</tr>
<tr>
<td>9</td>
<td>327.2–367.3</td>
<td>50.4</td>
<td>101.5</td>
<td>4.49</td>
<td>118.4</td>
<td>4.71</td>
</tr>
<tr>
<td>10</td>
<td>177.9–210.7</td>
<td>22.7</td>
<td>97.9</td>
<td>1.08</td>
<td>122.1</td>
<td>1.08</td>
</tr>
</tbody>
</table>
FIGURES

FIG. 1. Contour plots of the density in the reaction plane for the collision $^{100}\text{Mo} + ^{120}\text{Sn}$ at 14 $MeV/A$, at three different impact parameters: (a) $b = 8.5$ $fm$, (b) $b = 9$ $fm$, and (c) $b = 10$ $fm$. The size of the box is 40 $fm$.

FIG. 2. The average number of evaporated nucleons for the collision $^{100}\text{Mo} + ^{120}\text{Sn}$ at 14 $MeV/A$ as a function of the primary mass of the PLF fragment, in the direct (crosses) and reverse (open circles) kinematics: (a) Experimental results for $TKEL = 300 - 350$ $MeV$, Ref. [2]; (b) Our calculations for $b = 9$ $fm$, that corresponds to $TKEL = 325 - 370$ $MeV$. The full (dashed) line fits the points obtained in the direct (reverse) kinematics.