Solar opacity, neutrino signals and helioseismology

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Abstract

In connection with the recent suggestion by Tsytovich et al. [1] that opacity in the solar core could be overestimated, we consider the following questions:
i) What would a 10% opacity reduction imply for the solar neutrino puzzle?
ii) Is there any hope of solving the solar neutrino puzzle by changing opacity?
iii) Is a 10% opacity reduction testable with helioseismological data?

Recently Tsytovich et al. [1] reviewed corrections to the theory of photon transport in a dense hot plasma and claimed that previous calculations overestimate solar opacity $\kappa$ near solar center by about 10%. More generally the accuracy of the calculated opacity in the solar core is controversial: the characteristic difference between the calculations of the Livermore and Los Alamos groups is about (2–5)% [2] and essentially on these grounds Bahcall and Pinsonneault [3] estimate a (1σ) uncertainty of about 2.5%. On the other hand, Turck-Chièze et al. [4] argued the uncertainty to exceed 5%, a point criticized in [3]. It is worth recalling that, for the conditions of the solar interior, no experimental check is available and the estimates of accuracy originate essentially from theoretical arguments.

In this short note we consider thus the following questions:
i) What would 10% opacity reduction imply for the solar neutrino puzzle?
ii) Is there any hope of solving the solar neutrino puzzle by changing opacity?
iii) Is a 10% opacity reduction testable with helioseismological data?

In our analysis we assume a uniform reduction of opacity, i.e. $\kappa(\rho, T, ...) \rightarrow x \kappa(\rho, T, ...)$, where the factor $x$ is independent of density, temperature and chemical composition (other approaches to the effect of solar opacity on neutrino fluxes are reported, for instance, in refs. [5–7]). The assumption of uniform opacity variations is clearly ad hoc, to restrict the possible solar models. Nevertheless, it can be used as a first attempt to analyze the effects of the Tsytovich et al. proposal, for the following reasons:
1) the plasma effect advocated in [1] are mainly governed by the quantity $\delta = c^2 \omega^2_{pl} / v_T^2 \omega^2$, see [1] for notations. As $\delta \propto n_e / T^3$, it stays approximately constant along the solar profile.

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2) Assuming the plasma effect to be the same as in the solar center, the consequences are presumably overestimated, an attitude which is useful to tell if the solar neutrino puzzle can be solved by changing opacity.

As a first step, we compare the results of solar neutrino experiments with the predictions of our Standard Solar Model (SSM) and with those of a solar model with opacity reduced by 10%, see Table I. Our Standard Solar Model is obtained with the most updated version of FRANEC where diffusion of Helium and heavy elements is included [8].

Essentially, with respect to the SSM, one has a 20% reduction in the Chlorine and Kamiokande signals, and a 6% reduction of Gallium signal. The comparison with experimental results shows that this reduced opacity model is still inconsistent with data ($\chi^2/d.o.f. \approx 74$).

To explore the possibility of solving the solar neutrino puzzle by playing with opacity, we decreased it down to $x \approx 0.6$, see again Tab. I and Fig. 1. Even a variation of opacity well beyond the theoretical uncertainties cannot solve the solar neutrino puzzle: the best fit is obtained for $x = 0.64$ but it corresponds to $\chi^2/d.o.f. \approx 10.8$. The reason of the failure is that one cannot reproduce simultaneously the $^7\text{Be}$+CNO and $^8\text{B}$ neutrino fluxes implied by experimental results, in the assumption of standard neutrinos, see again Fig. 1.

To understand what is going on, we remark that a less opaque Sun is essentially a cooler Sun [9,10], with practically the same profile of density and pressure as the SSM; on the other hand the temperature profile has the same shape as that of the SSM, with a changement of scale, by the factor:

$$T_c/T_c^{SSM} = x^{0.14}.$$  \hspace{1cm} (1)

(These homology relationships are accurate to the 1% level or better for any quantity characterizing the solar structure, throughout all the radiative region).

The dependences of neutrino fluxes on central temperature are well known [10–13], by defining $\Phi_1 = \Phi_1^{SSM} (T_c/T_c^{SSM})^{\beta_i}$ we found (for our model with diffusion):

$$\beta_p = -0.7 \quad \beta_{Be} = 8.8 \quad \beta_{CNO} = 13.6 \quad \beta_B = 19.2.$$  \hspace{1cm} (2)

By using eq.(1) one can derive the dependence on the opacity parameter $x$. By writing $\Phi_1 = \Phi_1^{SSM} x^{\alpha_i}$ one gets:

$$\alpha_p = -0.1 \quad \alpha_{Be} = 1.2 \quad \alpha_{CNO} = 1.9 \quad \alpha_B = 2.7,$$  \hspace{1cm} (3)

which are in excellent agreement with the numerical results.

As well known, solar models with reduced central temperature cannot reconcile theory and experiments, see e.g. refs. [9,10,14], and thus it is not a surprise that an opacity variation, however large, cannot solve the solar neutrino problem.

In recent years, helioseismology has provided challenging tests of the Standard Solar Model. The values of the Helium mass fraction and of the mixing length, which were free parameters for solar model builders before the advent of helioseismology, are now strongly constrained by helioseismological determinations of the bottom of the convective zone ($R_b$ and $c_b$) and of the photospheric Helium content ($Y_{\text{photo}}$). By taking into account theoretical uncertainties (e.g. from equation of state and/or different inversion method), helioseismology gives:
\[ R_b/R_\odot = 0.710 - 0.716 \quad ; \quad c_b = (0.221 - 0.225) \text{ Mm/s} \quad ; \quad Y_{\text{photo}} = 0.233 - 0.268. \quad (4) \]

Actually, within the present uncertainties, the information on \( R_b \) and on \( c_b \) are not independent, since to the per cent level, \( c_b \) is related to \( R_b \) through \[ (5) \]

\[ c_b^2 \simeq \frac{2 GM_\odot}{3 R_\odot} \left( \frac{R_\odot}{R_b} - 1 \right) \]

Concerning \( Y_{\text{photo}} \) we have reported in eq. (4) the total range of published helioseismological determinations, see \[ (11) \]. It is important that only standard solar models including diffusion of helium and heavy elements satisfy these helioseismological constraint, whereas other models fails, see Tab. \[ II \] and Fig. \[ 2 \].

Starting from our SSM, we studied the effect of varying the opacity. As shown in Fig. \[ 2 \], the depth of the convective zone is weakly sensitive to such variations, whereas the photospheric helium abundance is sensibly related to opacity. A ten per cent reduction of this latter brings \( Y_{\text{photo}} \) well below the range allowed by helioseismology.

In conclusion, we would like to remark the following points:

i) Opacity can be tested to the ten per cent level or better by helioseismology (note that we have used only a subset of the helioseismological information, since helioseismology determines the sound speed also well below the bottom of the convective zone).

ii) The disagreement between helioseismology and the \( x = 0.9 \) solar model is not necessarily a proof against the Tsytovich \textit{et al.} proposal, as in this case the behaviour of opacity along the solar profile can be crucial.

iii) Last but not least, there is no hope of solving the solar neutrino puzzle by playing with opacity.

**ACKNOWLEDGMENTS**

We are extremely grateful to V. Berezinsky and G. Fiorentini for suggesting the problem as well as for their useful suggestions, and to J. Bahcall and J. Christensen-Dalsgaard for interesting comments. We express our gratitude to S. Degl’ Innocenti and F. Ciacio for the collaborative effort in updating FRANEC.
REFERENCES

[16] For the Gallium signal we use the weighted average of Gallex [17] and Sage [18] results.
TABLE I. Experimental results [16,19,20], predictions of our SSM and those of solar models with opacity reduced by the multiplicative factor $x$ (i.e. $\kappa \rightarrow x\kappa$). The values of $\chi^2$ per degree of freedom, calculated including only experimental errors, are also shown.

<table>
<thead>
<tr>
<th></th>
<th>EXP</th>
<th>SSM</th>
<th>$x = 0.9$</th>
<th>$x = 0.8$</th>
<th>$x = 0.7$</th>
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<tbody>
<tr>
<td>$S_{Cl}$ [SNU]</td>
<td>2.55 ± 0.25</td>
<td>7.4</td>
<td>5.9</td>
<td>4.5</td>
<td>3.4</td>
</tr>
<tr>
<td>$S_{Ga}$ [SNU]</td>
<td>74 ± 8</td>
<td>128</td>
<td>120</td>
<td>113</td>
<td>107</td>
</tr>
<tr>
<td>$\Phi_B$ $[10^6$/cm/s]</td>
<td>2.73 ± 0.38</td>
<td>5.16</td>
<td>3.92</td>
<td>2.83</td>
<td>1.96</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f.</td>
<td>154</td>
<td>74</td>
<td>28</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

TABLE II. The depth of the convective zone and the photospheric helium abundance as determined from helioseismology and as predicted by recent standard solar models.

<table>
<thead>
<tr>
<th></th>
<th>$R_b/R_\odot$</th>
<th>$Y_{photo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HELIOSEISMOLOGY</td>
<td>0.710–0.716</td>
<td>0.233–0.268</td>
</tr>
<tr>
<td><strong>SSMs with diffusion of He and Z</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRANEC96 [8]</td>
<td>0.716</td>
<td>0.238</td>
</tr>
<tr>
<td>BP95 [21]</td>
<td>0.712</td>
<td>0.247</td>
</tr>
<tr>
<td>P94 [22]</td>
<td>0.712</td>
<td>0.251</td>
</tr>
<tr>
<td>RVCD96 [23]</td>
<td>0.716</td>
<td>0.258</td>
</tr>
<tr>
<td><strong>SSMs with diffusion of He</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BP92 [3]</td>
<td>0.707</td>
<td>0.247</td>
</tr>
<tr>
<td>P94 [22]</td>
<td>0.710</td>
<td>0.246</td>
</tr>
<tr>
<td>BCDSTT [24]</td>
<td>0.707</td>
<td>0.248</td>
</tr>
<tr>
<td><strong>SSMs without diffusion</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FRANEC96-ND [8]</td>
<td>0.728</td>
<td>0.261</td>
</tr>
<tr>
<td>BP95-ND [21]</td>
<td>0.726</td>
<td>0.268</td>
</tr>
<tr>
<td>P94-ND [22]</td>
<td>0.726</td>
<td>0.270</td>
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<tr>
<td>RVCD96-ND [23]</td>
<td>0.725</td>
<td>0.278</td>
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<tr>
<td>BCDSTT [24]</td>
<td>0.721</td>
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</tr>
<tr>
<td>TCL [3]</td>
<td>0.725</td>
<td>0.271</td>
</tr>
</tbody>
</table>
FIGURES

FIG. 1. The region within $2\sigma$ from each experimental result [16,19,20] for standard neutrinos (dashed area), the prediction of our SSM (diamond) with estimated $1\sigma$ errors (bars) and the predictions for different opacity reductions (crosses). The analytical estimate (dotted curve) is also shown.

FIG. 2. The photospheric helium mass fraction $Y_{\text{photo}}$ and the depth of the convective zone ($R_b/R_\odot$):

a) as constrained from helioseismology (the dotted rectangle)

b) as predicted by solar models without diffusion (open circles), with helium diffusion (full squares) and with helium and heavy elements diffusion (full circles and diamond). Also shown are the effects of varying opacity, for the indicated values of $x$. 
Fig. 1
Fig. 2