We investigate the possibility that electromagnetic fluctuations are amplified in expanding universe by parametric resonance, during the oscillatory regime of a scalar field to which they are coupled. We consider scalar electrodynamics and we find that electromagnetic fluctuations undergo exponential instabilities. This mechanism could have some relevance for the problem of large scale primordial magnetic fields.

1 Introduction

The origin of $\mu$G galactic and intergalactic fields which are coherent over Kpc scales is still a puzzle for astrophysicists. These could be generated during galaxy formation or just amplified by a dynamo effect from primordial seeds of cosmological origin (see also O. Tornkvist in this volume).

The most important requirements to get reasonably agreement with observations concern the amplitude and the large coherence length of the magnetic fields observed. The amplitude of the magnetic field redshifts due to the flux conservation. Moreover, there is a causal upper bound to the scale of coherence at which the magnetic seed is generated, related to the horizon scale (and then to the Hubble scale $H^{-1}$ if there is not a mismatch between these two scales).

This latter condition is the most serious problem for all the microphysical mechanisms, such as thermal fluctuations in plasma, phase transition, just to cite few examples, and it resembles the large scale structure (LSS) problem for the Big Bang scenarios. The mechanisms produce easily a magnetic seed with an amplitude of the right order of magnitude, but on too small scales.

In a totally different perspective, one can try to address the problem of cosmological magnetic fields as the LSS problem was faced with inflation. Then the scale is no more a problem, but the amplitude becomes a serious issue. This fact is due to the conformal invariance of the electromagnetic field in conformally invariant space-times, such as the cosmological ones. For this reason electromagnetic fields do not suffer amplification by the time-dependent geometry, as minimally coupled scalar fields, gravitational waves do.
fore, the breaking of the conformal invariance was investigated as one of the necessary conditions for the generation of primordial magnetic fields by Turner and Widrow.\textsuperscript{5} By using alternative theories of gravity (where the electromagnetic field is not conformally coupled to a conformally flat geometry) magnetic seeds of the right order of magnitude can be achieved.\textsuperscript{6}

When the electromagnetic (EM) field is considered in interaction with other fields besides gravity –as with a charged scalar field for instance–, then its conformal invariant property is naturally broken because of the interaction term. The study of interacting quantum fields in cosmological spacetimes has received a renewed interest since it was realized that parametric resonance (PR) could play an important role.\textsuperscript{7} In the context of generation of cosmological magnetic fields a little attention was paid to the possibility that PR could play some effect in the interaction of the EM field with other fields. Even during a phase transition, the coherent oscillation of the order parameter around its minimum could enhance the production of fields coupled to it. In our opinion, there are four reasons in order to consider the effect of PR for EM fields:

a) the photon is massless, and this is almost a necessary condition in order to have appreciable amplification in expanding universe without considering very large couplings.\textsuperscript{8}

b) the effect due to PR for fields which would be conformally invariant without the interaction is less disputed by the expansion of the universe, because the equations of motion are conformally related to equations of motion in the Minkowski spacetime.\textsuperscript{9}

c) the growth due to parametric resonance is exponential as the one predicted by the dynamo effect,\textsuperscript{10} one of the astrophysical processes postulated to explain the observational evidence for galactic magnetic fields.

d) there are several examples in which the effect of PR is manifest on the maximum causal scale allowed by the problem (the coherence scale of the field). Since the scale of the magnetic seeds is always a crucial issue for a model which aims to explain their origin, this feature of PR is potentially very interesting.

\section{A scalar electrodynamics model}

We consider the Abelian-Higgs model in Robertson-Walker background\textsuperscript{11}

\begin{equation}
\mathcal{L} = \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - (D_{\mu}\Phi)^*(D^{\mu}\Phi) - V(\Phi^{*}\Phi)
\end{equation}
By working in the Coulomb gauge, the equation of motion for Fourier component of the transverse fluctuations of the gauge field are
\[ A''_{Tk} + (k^2 + 4\pi^2 a^2 \rho^2) A_{Tk} = 0, \]  
where a prime denotes derivative with respect to the conformal time \( \eta \) and \( \rho \) is implicitly defined as \( \Phi(t) = e^{i\theta(t)} \rho(t)/\sqrt{2} \) and satisfies
\[ \ddot{\rho} + 3H \dot{\rho} + \frac{\partial V}{\partial \rho} = 0, \]  
where we have neglected any back-reaction term. The non-dynamical quantity \( A_{0k} \) is given by the equation analogous to the Gauss law.

Equation (2) describes a harmonic oscillator with time dependent frequency: during the oscillation of the complex scalar field Eq. (2) can be reduced to a Mathieu-like equation. The solutions to this type of equation show an exponential instability \( \propto e^{\mu_k \eta} \), for some interval of frequencies, called \textit{resonance bands}. The structure of the resonance bands depends strongly on the time behaviour of the homogeneous scalar field, which on turn depends on the form of the potential \( V(\Phi) \). An example of stable resonance on long wavelengths is given by \( V = \lambda (\Phi^*\Phi)^2 \) with \( 4\pi^2 = 2\lambda \). The physical fields and the energy density \( T_{00}^{\text{EM}} \) are obtained through
\[ E = \frac{1}{a} (\nabla A_0 - A') \quad B = \frac{1}{a} \nabla \times A \quad T_{00}^{\text{EM}} = \frac{1}{8\pi a^2} (E^2 + B^2) \]

3 Discussions and Conclusions

The influence of plasma effects on the amplification driven by a coherent condensate is discussed in Ref. [11]: generically, the presence of other not coherent charges counteracts the resonance. However, one can think that in preheating after inflation, when it is assumed that the "creation" of charged particles is contemporaneous to the amplification of EM fluctuations, the resonance could proceed just as worked out in the vacuum case. In this way a charged inflaton could play the role of the condensate. If the resonance occurs on long wavelengths, then this would be on observable scales, because of the coherence of the inflaton on super-Hubble scales [12].

This would lead to an additional growth of the fraction of the electromagnetic energy density relative to the total one during reheating, which is missed in the usual predictions based on inflation-inspired models:
\[ r_{\text{reh}} \equiv \frac{\rho_\gamma}{\rho_{\text{tot}}} \bigg|_{\text{reh}} = r_{\text{infl}} \times \begin{cases} e^{2\mu_k \eta}/\eta^2 & \text{for } V = m^2 \Phi^*\Phi \\ e^{2\mu_k \eta} & \text{for } V = \lambda (\Phi^*\Phi)^2 \end{cases} \]
The interpretation of this growth of electromagnetic fluctuations as magnetic fields relies on the same idea used for inflation and string-produced magnetic fields \cite{5,6}: on large scale the large conductivity suppresses the electric part, and the magnetic part evolves along with magnetic flux conservation.

Acknowledgments

This work is in collaboration with A. Gruppuso. We thank the organizer of COSMO-99 for a stimulating and enjoyable conference and R. Brandenberger, A.-C. Davis, K. Dimopoulos, A. Riotto, L. Sorbo, O. Tornkvist comments, discussions and questions.

References