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Stochastic resonance in climatic change

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ABSTRACT

An amplification of random perturbations by the interaction of non-linearities internal to the climatic system with external, orbital forcing is found. This stochastic resonance is investigated in a highly simplified, zero-dimensional climate model. It is conceivable that this new type of resonance might play a role in explaining the 10^5 year peak in the power spectra of paleoclimatic records.

1. Introduction

The dominant feature of quaternary climate records is the 10^5 year peak in their power spectrum (Hays et al., 1976), which corresponds roughly to the alternation between glacial and interglacial stages. During the last few years many attempts have been made to clarify whether this peak is due mainly to external causes, such as variations of the insolation, or to internal mechanisms, such as oceanic and atmospheric feedbacks or volcanic eruptions.

Energy balance models (EBMs) are a useful tool in approaching the problem. These are the simplest possible models of the climatic system capable of incorporating some of the physical mechanisms believed to play a role in the time scales of interest. Those versions of EBMs studied heretofore exhibited some remarkable climatic properties such as multiple state equilibria. They failed, however, to explain the 10^5 year peak. We present in Fig. 1 a typical power spectrum of paleoclimatic variations for the last 700 000 years. A strong peak is present at a periodicity of 10^5 years, while smaller peaks can be noted at periods of 4×10^4 and 2×10^4 years.

As suggested by Milankovich (1930), such frequencies could be related to variations in the earth's orbital parameters. It appeared plausible therefore that causes of climatic variations should be associated with this external astronomical forcing. Studies using energy-balance models were able to reproduce the smaller 4×10^4 and 2×10^4 peaks when including such a forcing. However, no response is present which would correspond to the 10^5 year cycle. Hasselman (1976) pointed out the general possibility of short-time scale phenomena, modelled as stochastic perturbations, affecting long-term climate variations. Sutera (1981) (hereafter called S) has shown specifically that including such stochastic perturbations into an energy-balance model without deterministic external forcing, could lead to random transitions between the equilibria of the model. These transitions, it was shown, could have an average characteristic time of the order of 10^5 years. The interpretation of two stable model equilibria as a glacial and interglacial climate was suggested. Similar ideas have been suggested by Nicolis and Nicolis (1981). This left open the question about the transitions between the two being actually periodic, with period 10^5 years, as indicated by paleoclimatic records.

The purpose of this article is to investigate the interaction between the effect of a small external periodic forcing (about 0.1% of the solar constant) with a period of 10^5 years and the long-term effect of the random noise. The major conclusion is that

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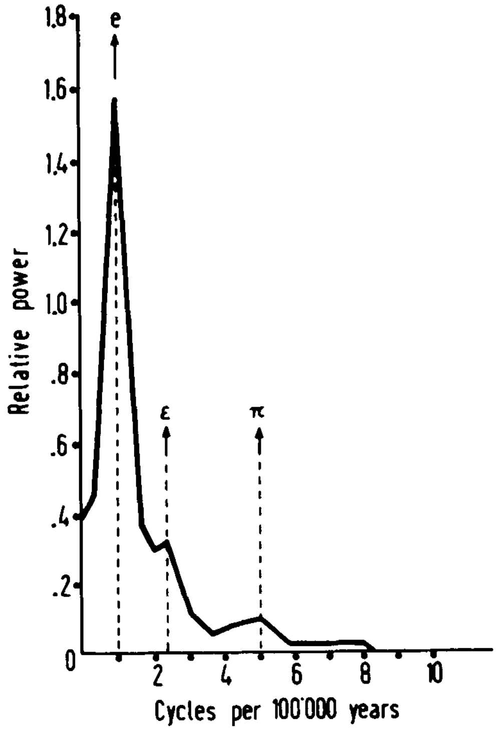

Fig. 1. Power spectrum of a time series of observation of the oxygen isotope content of fossil plankton in a deep-sea core from the equatorial Pacific which indicates fluctuations in global ice volume over the last 700 000 years. (From U.S. Committee for GARP, 1975, Academy Report, "Understanding climatic change", 1975, p. 144). The marked peaks have been tentatively associated with secular variations in the earth's orbit about the sun, namely changes in the eccentricity of the orbit (e), in the obliquity of its axis (ε) and the precession of the longitude of the perihelion (π).

our model, thus modified, does reproduce satisfactorily the sought-for periodicity. In Sections 2 and 3 we discuss the model used and in Section 4 we present our major results. Conclusions follow in Section 5.

2. A stochastically perturbed, zero-dimensional Budyko–Sellers model

In this section we shall present the model used here to study the effect of changes in the annually averaged solar radiation on the global earth temperature T. Our starting point is the usual, deterministic energy-balance model

$$C\frac{\mathrm{d}T}{\mathrm{d}t} = R_{\mathrm{im}}(T) - R_{\mathrm{out}}(T). \tag{1}$$

Here C is the thermal capacity of the earth, R_{in} is the incoming solar radiation and R_{out} the outgoing radiation. The parameterizations for R_{in} and R_{out} are:

$$R_{\mathrm{in}}(T) = Q\mu \tag{2a}$$

$$R_{\mathrm{out}}(T) = \alpha(T)Q\mu + \varepsilon(T) \tag{2b}$$

where $\varepsilon(T)$ is the long-wave surface radiation, $\alpha(T)$ the globally averaged albedo and Q is a long period average of incoming solar radiation. The dimensionless parameter μ will allow us to introduce an explicit variation in the solar input.

In general we can write eq. (1) in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}T = F(T)$$

where

$$F(T) = (R_{\mathrm{in}} - R_{\mathrm{out}})/C.$$

The solutions of the equation

$$F(T) = 0$$

represent steady states of eq. (1). These solutions define the "climates" of our model. Such a model "climate" is physically observable in paleoclimatic records if it is a *stable* steady state of eq. (1). To investigate the stability properties of climates we introduce a function Φ, hereafter called the pseudopotential, by

$$\Phi = -\int F(T)\,\mathrm{d}T. \tag{3a}$$

It is clear that

$$F(T) = -\frac{\partial\Phi}{\partial T}. \tag{3b}$$

Therefore the maxima and minima of Φ correspond to climates as previously defined. It is easy to show that the minima of Φ are unstable solutions. A climate will be called *observable* if it is a stable steady-state solution of eq. (1), i.e., a minimum of Φ.

We summarize the observational evidence in the following two statements.

(a) Climatic changes appear confined to a temperature range of a few degrees, say 10 K.
(b) Apart from the dramatic changes that happen on time scales of 10^4–10^5 years, temperatures seem to oscillate around fixed values.

In an energy-balance model, these considerations suggest the hypothesis of two stable climates separated by an unstable one in a range of temperature of about 10 K. Supportive evidence for our hypothesis is provided by the results of the one-dimensional energy-balance model of Bhattacharya and Ghil (1978) (see also Ghil and Battacharya, 1979). They obtained the stable–unstable–stable steady-state configuration over a 10 K temperature range, using a crude parameterization of the albedo which included certain conjectured cloud effects.

For illustrative purposes, we consider a simplified, zero-dimensional model based directly on the remarks above (see Section 3 for details).

$$\frac{dT}{dt} = \frac{\varepsilon(T)}{C} \times \left\{ \frac{\mu(t)}{1 + \beta[(1 - T/T_1)(1 - T/T_2)(1 - T/T_3)} - 1 \right\}. \tag{4}$$

Here $T_1 < T_2 < T_3$ are the three hypothesized climates, and μ is given by

$$\mu(t) = 1 + 0.0005 \cos \omega t$$

where

$$\omega = 2\pi/10^5 \text{ years}$$

The right-hand side of eq. (4) is now a function $\tilde{F} = \tilde{F}(T; t)$ to which corresponds a pseudo-potential $\tilde{\Phi} = \tilde{\Phi}(T; t)$. The change in μ corresponds approximately to the annual mean variation in insolation due to changes in ellipticity of the earth's orbit.

As it stands, does the model reproduce the observed 10 K changes? The answer is no because the numerical results show changes of at most 1 K around either stable climate. The same answer is also given by more detailed, one-dimensional seasonally varying models (North and Coakley, 1979; Pollard et al., 1980; Schneider and Thompson, 1979; Suarez and Held, 1979). Therefore, we have to look for different physical mechanisms in order to explain the amplitude of the 10^5-year cycle.

It has been suggested repeatedly that the 10^5-year cycle can arise from an internal mechanism due to atmospheric and oceanic circulations (e.g. Ghil, 1980). For our purpose, the result outlined in S is particularly relevant. In that paper, the following stochastic differential equation was considered:

$$\frac{dT}{dt} = F(T) + \sigma\eta(t). \tag{5}$$

Here $\eta(t)$ is a normalized Wiener process, commonly called white noise (see S for details) with mean zero and variance one. The noise simulates the global effect of relatively short-term fluctuations in the atmospheric and oceanic circulations on the long-term temperature behaviour. It was assumed that $F(T)$, like $\tilde{F}(T, t)$ in eq. (4), has two stable states, T_1 and T_3, with a temperature difference between them of about 10 K as shown in Fig. 2a. The important effect of the noise is that, starting from one of the two steady states, the solution of eq. (4) will jump after a long but *finite time* to the other steady state. The jumping time itself is a random variable.

The noise by itself cannot explain the 10^5-year cycle. Indeed, the temperature spectrum of T in a model governed by (5), with the pseudo-potential Φ of Fig. 2a, is just exponentially decaying (Fig. 2b). The theoretical reason for this is that the variance of the jumping time is approximately equal to the mean jumping time (for details see Gihman and Skorohod, 1972).

Let us now summarize the situation. With μ alone varying periodically by 0.1% over 10^5 years in the model (1, 2), only a very low amplitude periodic response in the temperature behaviour obtains. On the other hand, with a noise acting in climate model (5), but with fixed μ, large amplitude long-term temperature variations obtain, but no dominant periodicity is apparent. We shall now investigate both effects simultaneously, that is, we shall investigate the effect of the noise acting together with a periodic change of the solar constant.

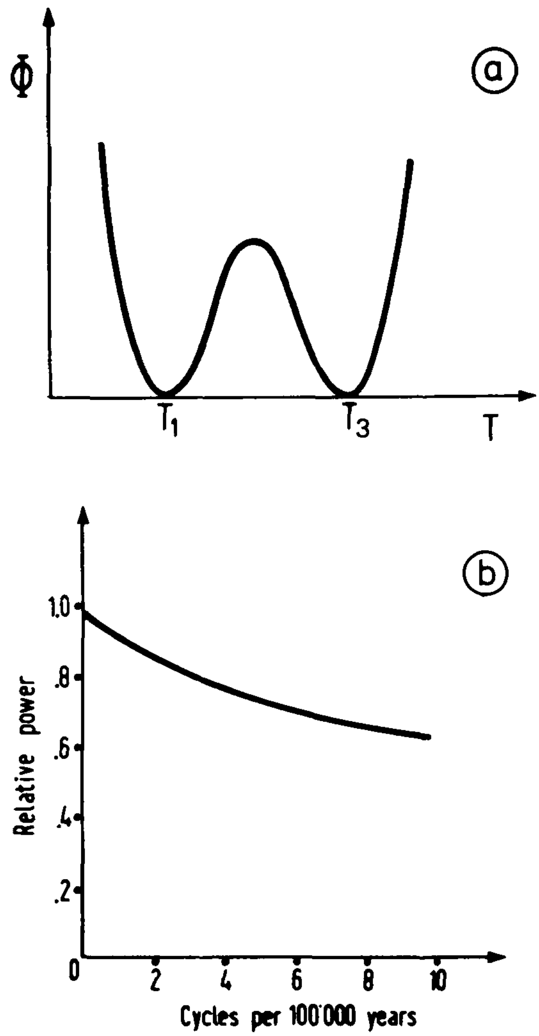

Fig. 2. (a) The pseudo-potential behaviour as assumed by Sutera (1981). The existence of two stable steady-state solutions of eq. (1) separated by about 10 K is reproduced using the model of Ghil and Bhattacharya (1979). (b) The power spectrum of the temperature T solution of eq. (5) with $\Phi(T)$ given in (a). Note that there is no evidence of periodicity.

3. The model equation

We present here the details of the model used in this article. From eq. (1) and the parameterization (2) of R_{in} and R_{out}, the steady states, or climates of our energy-balance model satisfy the equation

$$\mu(1-\alpha(T))Q=\varepsilon(T).$$

Let us define the function $f(T)$ by

$$f(T)=\frac{\varepsilon(T)}{Q(1-\alpha(T))}.$$

Clearly the "climates" of (1, 2) are the solutions of

$$f(T)=1.$$

Around a climate, therefore, we can always write $f(T)$ in the following way

$$f(T)=1+\delta(T).$$

The steady solutions of eq. (1) therefore satisfy the equation

$$\delta(T)=0.$$

Let us now introduce our main assumption, i.e., that the solutions are two stable steady states T_1, T_3, and one unstable state T_2 between them. Therefore, we can write $\delta(T)$ in a first approximation as

$$\delta(T)=\beta\left(1-\frac{T}{T_1}\right)\left(1-\frac{T}{T_2}\right)\left(1-\frac{T}{T_3}\right).$$

The dimensionless parameter β will be computed below. Using the equations above we have

$$(1-\alpha(T))Q=\frac{\varepsilon(T)}{1+\beta(1-T/T_1)(1-T/T_2)(1-T/T_3)}. \qquad (6)$$

We can now rewrite our model using eq. (6)

$$\frac{\mathrm{d}T}{\mathrm{d}t}=\tilde{F}(T;t)=\frac{\varepsilon(T)}{C}\times\left[\frac{\mu(t)}{1+\beta(1-T/T_1)(1-T/T_2)(1-T/T_3)}-1\right]. \qquad (7)$$

Note that $\mu=\mu(t)$ is now time dependent. Hereafter we shall assume

$$\mu(t)=1+0.0005\cos\omega t$$

where

$$\omega=2\pi/10^5 \text{ years}.$$

The dimensionless parameter β can now be computed as a function of the decay time τ near the actual present interglacial climate T_3. In other words, $d\tilde{F}/dT|_{T=T_3} = 1/\tau$ is given as a function of β for $\mu(t) = 1$, from (7):

$$\left.\frac{d\tilde{F}}{dT}\right|_{T=T_3} = \tau^{-1} = \frac{\varepsilon(T_3)}{C}\frac{\beta}{T_3}\left(1-\frac{T_3}{T_1}\right)\left(1-\frac{T_3}{T_2}\right). \quad (8)$$

Using a value of τ and C as given by GCM models, we can compute β from (8). In fact, we have done just that, using, however, the value of τ obtaining from the Ghil (1976) model.

4. Results

To understand clearly the effect of insolation variations on a stochastically driven model, we have to look at the behaviour of the pseudo-potential $\tilde{\Phi}(T; t)$, defined in section 3, as $\mu(t)$ is varying periodically with an amplitude of 0.1%. In Fig. 3 we show schematically the periodic behaviour of $\tilde{\Phi}(T; t)$ for our model. Let us note that this behaviour is common to all energy-balance models that admit two observable climates separated by an amplitude of about 10 K. As we can see, the positions of the stable states themselves (abscissa) change only by a few tenths of a degree. The pseudo-potential difference between a stable state and the unstable state (ordinate) changes by a factor two over a period. The latter observation is the central point for our investigation.

It is possible to show from the theory of stochastic differential equations that, for instance, the jumping time from T_3 to T_1 for the pseudo-potential $\Phi(T)$ in Figs. 3a, 3e (at fixed t) is proportional to the *square* of the jumping time from T_3 to T_1 for the case of Fig. 3c. In other words, always referring to Fig. 3, let us consider a model starting from T_3 at $t = 0$ (Fig. 3a). Initially the probability of a jump is very small (nearly 0); it grows with time and at $t = 50\,000$ years (Fig. 3c) will be nearly 1. Hence, in most cases, a jump from T_3 to T_1 will actually occur. Now the probability of a reverse jump, from T_1 back to T_3, is nearly zero. This probability, however, increases with time, and at $t = 100000$ years (Fig. 3e) it becomes nearly 1, leading in most cases to a jump from T_1 to T_3. This variation of probabilities in time repeats itself with period 10^5 years. The probability of a jump, from T_1 to T_3 or from T_3 to T_1, depends *exponentially* on the noise level. Hence, its approaching closely both 0, for the maximum depth of the pseudopotential well (at T_3 in Figs. 3a, 3e and at T_1 in Fig. 3c), and

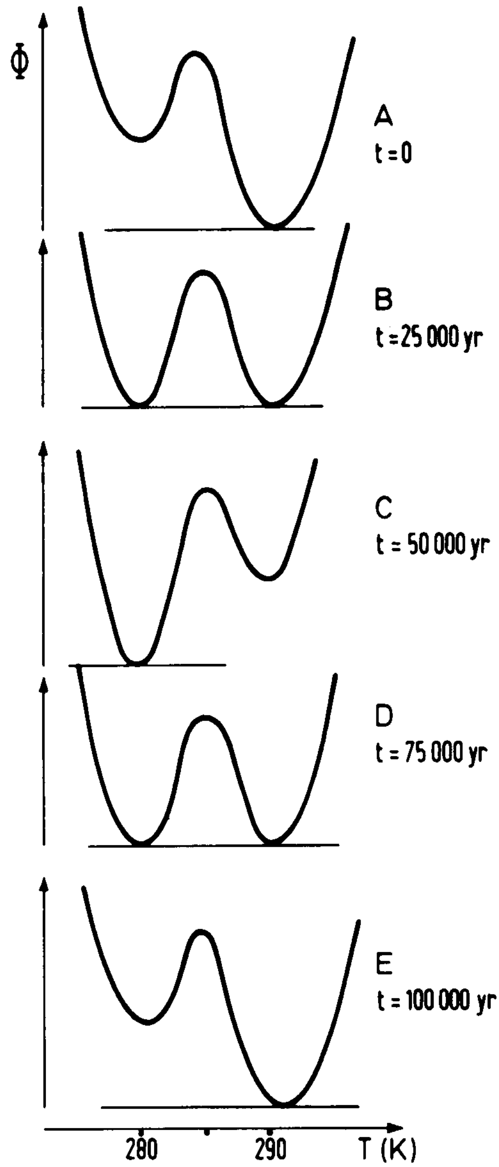

Fig. 3. The behaviour of the pseudo-potential $\Phi(T; t)$ as a function of time for a model of the Ghil–Bhattacharya (1979) type. Note that the maxima and minima of $\Phi(T; t)$ are changing only by a few tenths of degree during the cycle, while the pseudo-potential difference between the stable points and the unstable point is changing by a factor of two.

1, for the minimum depth (T_1 in Figs. 3a, 3e and T_3 in Fig. 3c) will occur only for a given range of the noise variance σ^2.

In Fig. 4, we show an individual realization of a solution to the model, in which $T_3 - T_1 = 10$ K. Fig. 5 shows the corresponding power spectrum. The variance used for this particular simulation was about 0.15 K^2/year. This value is compatible with the present estimates of the historically observed global temperature fluctuations (see, for instance Budyko, 1969). Clearly, the temperature behaviour is periodical, shifting from T_3 to T_1, and from T_1 to T_3, every 10^5 years, with small gaussian oscillations around the stable states. This is our main result. Similar results were obtained by Nicolis (1980), using the Fokker–Planck equation derived from eq. (5). The same qualitative behaviour, with climate jumps from T_3 to T_1 and back every 10^5 years, was observed for a variance of the noise in the range 0.14–0.22 K^2/year. Inside this range our approach indicates the possibility of a new type of resonance, namely *stochastic resonance* between a deterministic external forcing of a climate model and a stochastic internal mechanism. The word resonance is appropriate because if the noise is too small there is no correlation between the jumping time and the periodic change of the insolation. The same is true if the noise is too strong.

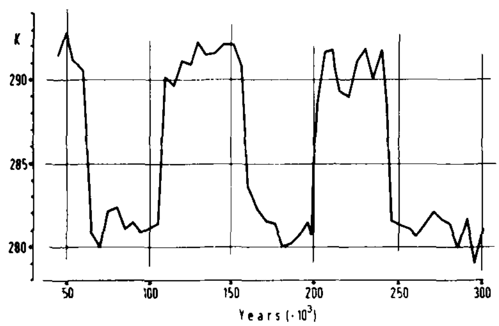

Fig. 4. Computer simulation of eq. (5) for heat-budget model with two observable climates at 280 and 290 K. The variance of the noise was about 0.15 K^2/year.

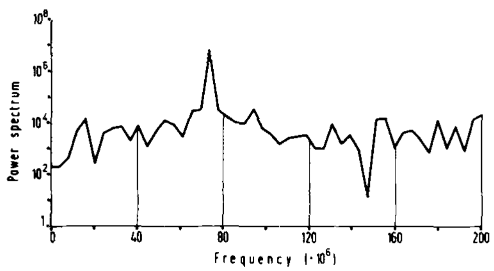

Fig. 5. The power spectrum of the solution shown in Fig. 4.

5. Conclusions

Our results point to the possibility of explaining large amplitude, long-term alternations of temperature by means of a co-operation between external periodic forcing due to orbital variations and an internal stochastic mechanism. The external periodic forcing alone is unable to reproduce the major peak in the observed quaternary climate records. The internal stochastic forcing alone does not reproduce it either. The combination of the two effects, however, produces what we may call a stochastic resonance, which amplifies the small external forcing: a small change in the external forcing induces a large change in the probability of jumping between two observable climates. This new mechanism could be useful in our understanding of long-term climatic change. At any rate, it seems to warrant further investigation.

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