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A pulsational approach to near infrared and visual magnitudes of RR Lyrae stars

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ABSTRACT

In this paper we present an improved theoretical scenario concerning near infrared and visual magnitudes of RR Lyrae variables, as based on up-to-date pulsating models. New relations connecting V and K absolute magnitudes with periods, mass, luminosity and metal content are discussed separately for fundamental and first overtone pulsators. We also show that the $V-K$ colors are predicted to supply tight constraints on the pulsator intrinsic luminosity. On this basis, we revisit the case of the prototype variable RR Lyr, showing that the parallax inferred by this new pulsational approach appears in close agreement with HST absolute parallax. Moreover, available K and V measurements for field and cluster RR Lyrae variables with known reddening and metal content are used to derive a relation connecting the K absolute magnitude to period and metallicity (M_K-[Fe/H]-log$P$) as well as a new calibration of the M_V-[Fe/H] relation. The comparison between theoretical prescriptions and observations suggests that RR Lyrae stars in the field and in Galactic Globular Clusters (GGCs) should have quite similar evolutionary histories. The comparison between theory and observations also discloses a general agreement that supports the reliability of current pulsational scenario. On the contrary, current empirical absolute magnitudes based on the Baade-Wesselink (BW) method suggest relations with a zero-point that is fainter than predicted by pulsation models, together with a milder metallicity dependence. However, preliminary results based on a new calibration of the BW method provided by Cacciari et al. (2000) for RR Cet and SW And appear in a much better agreement with the pulsational predictions.

Key words: globular clusters: distances – stars: evolution – stars: horizontal branch – stars: oscillations – stars: variables: RR Lyrae

1 INTRODUCTION

Cosmic distances are fundamental cornerstones in modern astronomy, and reliable distance indicators can supply tight constraints on several cosmological parameters (see e.g. Goobar et al. 2000; Ferrarese et al. 2000). Accordingly, the number of theoretical and empirical investigations looking for new standard candles and/or aimed at improving the classical ones is countless in the recent literature (see e.g. Salaris & Cassisi 1997; Bono et al. 1999; Feast 1999; Walker 1999; Caputo et al. 2000; Carretta et al. 2000; Castellani et al. 2000; Gieren, Fouqué, & Storm 2000; Layden 2002). However, the ongoing problems affecting distance determinations are soundly revealed by current uncertainties on the true distance modulus μ_0 of the nearest galaxy, the Large Magellanic Cloud (LMC), which is often taken as a reference for extragalactic distance determinations. In fact, current estimates range from $\mu_{0,LMC} \approx 18.37 \pm 0.23$ mag (RR Lyrae, Luri et al. 1998) to ≈ 18.7 mag (classical Cepheids, Feast & Catchpole 1997; Bono et al. 1999; Caputo et al. 2002).

In this paper we discuss RR Lyrae stars, the radial pulsators connected with low-mass He-burning stars in the Horizontal Branch (HB) evolutionary phase. The use of these variables as distance indicators appears still hampered by significant uncertainties, and indeed the relation between absolute visual magnitude and iron content, M_V-[Fe/H], calibrated according to different empirical or theoretical constraints, supplies absolute magnitudes that for a given metallicity cover ≈ 0.3 mag. Oddly enough, one also finds that the empirical approach based on the BW method, gives different results when different bands (optical vs near infrared) or different approaches (surface brightness vs canonical BW) are used.

In this context, the endeavor to link RR Lyrae pulsation properties, i.e. periods and/or amplitudes of the light curve, to the absolute magnitudes of these stars (see Sandage 1993; Caputo 1997; Caputo et al. 2000; Caputo, Degl'Innocenti & Marconi 2001) appear of great relevance, because periods (and amplitudes) are firm and safe observational parameters, independent of distance and reddening. Empirical investigations (Longmore, Fernley, & Jameson 1986; Longmore et al. 1990) have already suggested that in the K band the RR Lyrae stars in GGCs follow a rather well-defined Period-Luminosity (PL_K) relation whose slope is, within observational errors, almost constant when moving from metal-poor to metal-rich clusters. This scenario has been recently supported by the theoretical investigation by Bono et al. (2001, Paper I). In that paper, on the basis of nonlinear, convective RR Lyrae models, we found that the K-magnitude of fundamental (F) and fundamentalized first-overtone (FO) pulsators can be fitted by a common Period-Luminosity-Metallicity (PLZ_K) relation which appears to be in good agreement with observations.

In this paper we present improved predictions concerning both F and FO pulsators. Section 2 presents new theoretical relations concerning V and K absolute magnitudes. On this basis, in the same section we discuss improved PLZ_K relations, showing that $V-K$ colors might allow us to constrain the intrinsic bolometric luminosity of the pulsators. Section 3 deals with the prototype variable RR Lyr, used as fundamental calibrator of the RR Lyrae distance scale, while observational data for field and cluster RR Lyrae stars are discussed in Section 4. In Section 5 we present a new relation connecting the K absolute magnitude to period and metallicity (M_K-[Fe/H]-log$P$) as well as a refined M_V-[Fe/H] relation. A few final remarks concerning both theoretical models and observations are outlined in Section 6

2 NEW PLZ_K AND PLC_{VK} RELATIONS FOR F AND FO PULSATORS

To properly implement the pulsational framework presented in Paper I, we computed additional pulsating models adopting a finer grid of effective temperatures. We ended up with 132 F and 91 FO nonlinear models that cover a wide metallicity range $0.0001 \leq Z \leq 0.02$. For each metal abundance the helium content was fixed according to a primordial helium abundance of Y_p=0.24 and a helium-to-metal enrichment ratio $\Delta Y/\Delta Z \approx 2.5$.

At fixed chemical composition, we explored three different luminosity levels that cover current uncertainties on the ZAHB luminosity as well as post-ZAHB evolutionary effects (see column 3 in Table 1). As for as the pulsator mass is concerned, we adopted for each given metallicity a "reference mass" M_r, roughly following the correlation between metal content and mass of ZAHB models populating the RR Lyrae instability strip, i.e. $3.77 \leq \log T_e \leq 3.86$. These M_r values are listed in Table 1 (column 4), together with the evolutionary mass (with an average uncertainty of $\sim 2\%$) of ZAHB models at logT_e=3.85 and 3.80, as inferred according to evolutionary computations available in the literature. In order to investigate the dependence of the predicted relations on the pulsator mass, additional models were finally constructed at selected metal contents (Z = 0.0001 and Z = 0.001) but with slightly different reference mass values.

Table 1. Input parameters for the grid of RR Lyrae models. Mass and luminosity are in solar units.

Z	Y	log L	M_r	$M(3.85)$ $\pm 2\%$	$M(3.80)$ $\pm 2\%$
0.0001	0.24	1.61, 1.72, 1.81	0.75	0.796	0.852
0.0004	0.24	1.61, 1.72, 1.81	0.70	0.699	0.721
0.001	0.24	1.51, 1.61, 1.72	0.65	0.648	0.666
0.006	0.255	1.55, 1.65, 1.75	0.58	0.585	0.589
0.01	0.255	1.51, 1.57, 1.65	0.58	0.575	0.578
0.02	0.28	1.41, 1.51, 1.61	0.53	0.545	0.546

The entire set of fundamental and first overtone nonlinear models were integrated in time till they approach the limit cycle stability. These calculations provided periods, pulsation amplitudes, and bolometric light curves. The light curves were transformed into the observational plane by adopting bolometric corrections and color-temperature relations provided by Castelli, Gratton & Kurucz (1997a,b). Tables 2 and 3 list the predicted periods (in days) and mean (magnitude-weighted) V and K magnitudes for the new F and FO pulsation models, respectively. These new data implement similar predictions already presented in Tables 2 and 3 of Paper I.

Data listed in Table 2 and Table 3 disclose that, for any given mass, luminosity and metal content, the visual magnitude M_V of the pulsators is quite independent of the effective temperature, whereas M_K shows a strong dependence on this parameter (see also Paper I). This effect is the consequence of the fact that in the range of RR Lyrae effective temperatures the visual bolometric correction is roughly constant, whereas in the K-band it is strongly dependent on the effective temperature. It follows that the K-band magnitudes become significantly brighter when moving toward cooler effective temperatures. The pulsation equation $P = f(L, M, T_e)$ connects, for each given chemical composition, periods to luminosity, mass and effective temperature. Therefore, the dependence of the absolute magnitude on T_e (through the BC) can be replaced with a dependence on period. Accordingly, when moving from the blue to the red edge of the instability strip the period increases at roughly constant absolute visual magnitude ($\delta M_V/\delta \log P \sim -0.14$), whereas M_K shows a much steeper correlation ($\delta M_K/\delta \log P \sim -2.1$.)

On the other hand, when varying the luminosity for any given value of mass and effective temperature, one has again a relation between absolute magnitude and period. The key feature of the K-band is that the two derivatives $(\delta M_K/\delta \log P)_{M,L)}$ and $(\delta M_K/\delta \log P)_{M,T_e)}$ are quite similar. In other words, one finds a sort of *degeneracy* for which the near-infrared period-magnitude (PL_K) relation is only marginally dependent on the bolometric luminosity of pulsators (see Fig. 1 in Paper I).

As for the dependence of the PL_K relation on the pulsator mass and metal content, current calculations confirm the results presented in Paper I. However, at variance with the approach adopted in that paper, where F and fundamentalised FO periods (i.e., $\log P_F = \log P_{FO} + 0.127$) were taken simultaneously into account, we found that the intrin-

Table 2. Periods and absolute mean (magnitude-weighted) magnitudes for the new fundamental models.

Z	$\log L/L_\odot$	T_e (K)	P (d)	M_K (mag)	M_V (mag)
0.0001	1.61	6800	0.3876	-0.172	0.848
0.0001	1.61	6700	0.4077	-0.226	0.845
0.0001	1.61	6600	0.4291	-0.282	0.844
0.0001	1.61	6500	0.4518	-0.328	0.834
0.0001	1.61	6400	0.4764	-0.378	0.833
0.0001	1.61	6200	0.5299	-0.470	0.843
0.0001	1.61	6100	0.5594	-0.507	0.848
0.0001	1.72	6800	0.4790	-0.448	0.597
0.0001	1.72	6600	0.5292	-0.554	0.572
0.0001	1.72	6400	0.5882	-0.658	0.562
0.0001	1.72	6200	0.6561	-0.752	0.566
0.0001	1.81	6600	0.6304	-0.778	0.357
0.0001	1.81	6400	0.7005	-0.880	0.338
0.0001	1.81	6300	0.7399	-0.930	0.335
0.0004	1.81	6600	0.6632	-0.778	0.330
0.0004	1.81	6500	0.7014	-0.830	0.320
0.0004	1.81	6400	0.7400	-0.881	0.315
0.006	1.75	6900	0.5988	-0.494	0.428
0.006	1.75	6800	0.6290	-0.546	0.426
0.006	1.75	6700	0.6614	-0.598	0.416
0.006	1.75	6600	0.6972	-0.650	0.408
0.006	1.75	6500	0.7349	-0.702	0.402
0.006	1.75	6400	0.7755	-0.753	0.399
0.006	1.75	6300	0.8184	-0.803	0.400
0.006	1.75	6200	0.8626	-0.850	0.403
0.006	1.75	6000	0.9632	-0.962	0.404
0.006	1.75	5900	1.0184	-0.988	0.423
0.006	1.75	5800	1.0794	-1.029	0.430
0.01	1.65	6800	0.5162	-0.300	0.697
0.01	1.65	6600	0.5708	-0.403	0.682
0.01	1.65	6500	0.6019	-0.455	0.672
0.01	1.65	6400	0.6322	-0.509	0.668
0.01	1.65	6300	0.6669	-0.558	0.665
0.01	1.65	6200	0.7052	-0.607	0.664
0.01	1.65	6100	0.7441	-0.658	0.668
0.01	1.65	5900	0.8345	-0.762	0.681
0.01	1.65	5700	0.9331	-0.839	0.695

Table 3. Periods and absolute mean (magnitude-weighted) magnitudes
for the new first overtone models.

Z	$\log L/L_\odot$	T_e (K)	P (d)	M_K (mag)	M_V (mag)
0.0001	1.61	7100	0.2546	-0.031	0.817
0.0001	1.61	7000	0.2657	-0.081	0.817
0.0001	1.61	6900	0.2784	-0.131	0.812
0.0001	1.61	6800	0.2915	-0.180	0.808
0.0001	1.61	6600	0.2935	-0.276	0.806
0.006	1.75	7000	0.4132	-0.453	0.325
0.006	1.75	6900	0.4349	-0.502	0.350
0.006	1.75	6800	0.4563	-0.552	0.363
0.006	1.75	6700	0.4806	-0.603	0.370
0.006	1.75	6600	0.5070	-0.656	0.374
0.006	1.75	6500	0.5343	-0.709	0.378
0.006	1.75	6400	0.5615	-0.759	0.383
0.006	1.75	6300	0.5904	-0.806	0.389
0.006	1.75	6200	0.6222	-0.849	0.394
0.01	1.51	6800	0.2871	0.045	0.965
0.01	1.57	6900	0.3070	-0.055	0.800
0.01	1.57	6800	0.3228	-0.104	0.807
0.01	1.65	7000	0.3394	-0.204	0.574
0.01	1.65	6900	0.3562	-0.253	0.594
0.01	1.65	6800	0.3745	-0.302	0.603
0.01	1.65	6700	0.3931	-0.355	0.611
0.01	1.65	6600	0.4135	-0.402	0.615

sic accuracy can be improved by independently treating the two pulsation modes.

By accounting also for the mass dependence, a linear regression through the models gives:

$$M_K^F = 0.511 - 2.102 \log P + 0.095 \log Z - 0.734 \log L - 1.753 \log M/M_r \quad (1)$$

$$M_K^{FO} = -0.029 - 2.265 \log P + 0.087 \log Z - 0.635 \log L - 1.633 \log M/M_r \quad (2)$$

where mass and luminosity are in solar units and the rms scatter is σ=0.016 mag. As already discussed in Paper I, the predicted slope at constant luminosity and metallicity appears in good agreement with empirical data for GGC variables, as given, e.g., by the RR Lyrae stars in M3 collected by Longmore et al. (1990).

The present enlarged set of pulsating models confirms that over the explored range of metallicity and luminosity the $M_K - \log P$ correlation is only mildly dependent on the luminosity, a variation of 0.1 dex implying a change of $\delta M_K \sim 0.07$ mag, as well as on the metallicity, a variation of 0.3 dex implying a change of $\delta M_K \sim 0.03$ mag. The predicted magnitudes are thus expected to be only mildly dependent on uncertainties introduced by these theoretical or observational parameters. On the other hand, the non negligible dependence on the pulsator mass should only introduce a marginal uncertainty in the error budget since we can use quite firm theoretical constraints on such evolutionary parameter. As a fact, HB models available in the recent literature supply slightly discordant luminosity values, but very similar predictions concerning the mass.

As well known, fundamental (*RRab*) and first-overtone (*RRc*) RR Lyrae populate the red and the blue side of the pulsation region, respectively. Therefore one may assume for F and FO pulsators the predicted stellar mass listed in Table 1 for ZAHB models at $\log T_e = 3.80$ and 3.85, respectively. The total uncertainty introduced by this assumption is of the order of 3.5% if we simultaneously account for the predicted spread in mass between F and FO pulsators and for the occurrence of post-ZAHB evolutionary effects.

By inserting these mass values into the previous relations, one eventually derives the following near-infrared Period-Luminosity-Metallicity (PLZ_K) relations:

$$M_K^F = 0.565 - 2.101 \log P + 0.125 \log Z - 0.734 \log L \quad (3)$$

$$M_K^{FO} = -0.016 - 2.265 \log P + 0.096 \log Z - 0.635 \log L \quad (4)$$

with a rms scatter of $\sigma_K = 0.031$ mag and 0.025 mag, respectively, including in quadrature an uncertainty of ~ 0.03 mag, as given by the adopted spread in mass (3.5%) at fixed metal abundance.

By following a similar procedure, one finds that also

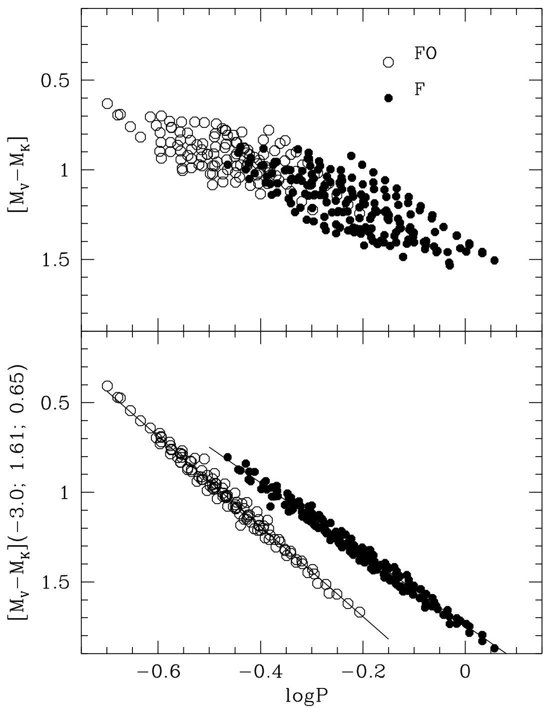

Figure 1. Top panel: predicted $M_V - M_K$ colors of F (filled circles) and FO pulsators (open circles) as a function of period (data in Table 2 and Table 3). Bottom panel: same as the top panel but with individual colors scaled to the same metallicity ($\log Z = -3$), luminosity ($\log L/L_\odot = 1.61$), and mass ($0.65M_\odot$). The solid lines display the predicted relations (see equation (5) and (6) in the text).

the T_e dependence of the pulsator color can be replaced by a period dependence. In particular, since the V magnitudes are strongly correlated with L, whereas the K magnitudes do not, we expect that the $V - K$ color is strongly dependent on luminosity, thus providing useful constraints to this unknown parameter.

As a fact, using the whole set of pulsating models, we derive:

$$(M_V - M_K)^F = 4.014 + 1.986 \log P - 0.134 \log Z - 1.662 \log L + 1.656 \log M/M_r \quad (5)$$

$$(M_V - M_K)^{FO} = 5.195 + 2.518 \log P - 0.159 \log Z - 2.158 \log L + 1.815 \log M/M_r \quad (6)$$

with a rms scatter of 0.027 mag and 0.023 mag, respectively (see Fig. 1).

By assuming the predicted masses at logT_e=3.80 and 3.85 for F and FO pulsators, respectively, the previous relations yield the Period-Luminosity-Color relations (PLC_{VK}) in the V, K bands for both F and FO pulsators, as given by:

$$(M_V - M_K)^F = 3.963 + 1.986 \log P - 0.162 \log Z - 1.662 \log L \quad (7)$$

$$(M_V - M_K)^{FO} = 5.180 + 2.518 \log P - 0.168 \log Z - 2.158 \log L \quad (8)$$

with a total intrinsic dispersion of 0.037 mag and 0.031 mag, respectively, which accounts for the adopted spread in mass (3.5%) at fixed metallicity.

According to eqs. (7) and (8), one finds that accurate V, K photometric measurements of F and FO pulsators with known period and metal content can supply independent information on the intrinsic luminosity of the variable. Moreover, since $E(V - K)$=0.89A_V (Cardelli et al. 1989), if the visual extinction along the line of sight is known with an accuracy of ± 0.03 mag, then the luminosity might be evaluated with a (formal) accuracy of ∼ 0.03 dex.

3 THE CASE OF RR LYR ITSELF

As already discussed by Bono et al. (2002, Paper II), the predicted PLZ_K relation for F pulsators can be used to provide for the prototype variable RR Lyr a "pulsational" parallax that appears in close agreement with the absolute value recently measured by Benedict et al. (2002, hereafter B02) on the basis of new astrometric data collected with FGS 3, the interferometer on board of HST. In that paper, *we assumed* an intrinsic luminosity of RR Lyr in the range log$L/L_\odot$=1.65-1.80 to cover current uncertainties of HB models. On this basis, we derived M_K=−0.541±0.062 mag and a "pulsation" parallax π_{puls}=3.858±0.131 mas, in close agreement with the HST absolute value π_{abs}=3.82±0.20 mas.

According to the results presented in the previous section, one may revisit the case of RR Lyr, avoiding any assumption on the RR Lyr bolometric luminosity. Adopting log$P$=−0.2466 (Kazarovets, Samus, & Durlevich 2001), K=6.54 mag (Fernley, Skillen, & Burki 1993), [Fe/H]=−1.39±0.15 (Fernley et al. 1998a; Beers et al. 2000) and V=7.784 mag (Hardie 1955), equation (7) gives $\log L = 1.642 \pm 0.024 + 0.535 A_V$. Thus, for each adopted extinction correction, one *derives* directly from the observed $V - K$ color the luminosity value to be inserted into equation (3), without using any evolutionary predictions. Once M_K is determined from equation (3), both the intrinsic distance modulus ($\mu_0 = K - 0.11A_V - M_K$) and the absolute visual magnitude ($M_V = V - A_V - \mu_0$) can be easily determined.

Data listed in Table 4 show that the pulsation parallax presents a mild dependence on the adopted extinction correction. In particular, the top panel in Fig. 2 shows that when moving from A_V = 0.01 to 0.22 mag, the pulsation parallax (filled circles) decreases from 3.89 ± 0.07 to 3.78 ± 0.07 mas. However, current pulsational estimates remain well within the errors of the HST absolute parallax (dashed lines), and present a better formal accuracy ($\sigma_\pi/\pi \sim 3\%$ versus $\sim 5\%$). According to this evidence the pulsation parallaxes resemble direct astrometric measurements that are independent of interstellar absorption.

Concerning the RR Lyr predicted absolute magnitude, the middle and the bottom panel in Fig. 2 show that when moving from A_V=0.01 to 0.22 mag M_K changes from -0.513 ± 0.041 mag to -0.595 ± 0.041 mag, while M_V from 0.722 ± 0.041 to 0.453 ± 0.041 mag, in close agreement with HST results (open circles), but with a better formal accuracy (±0.041 versus ±0.114 mag). In other words, despite the substantial improvement in the accuracy of the RR Lyr trigonometric parallax provided by HST, when compared

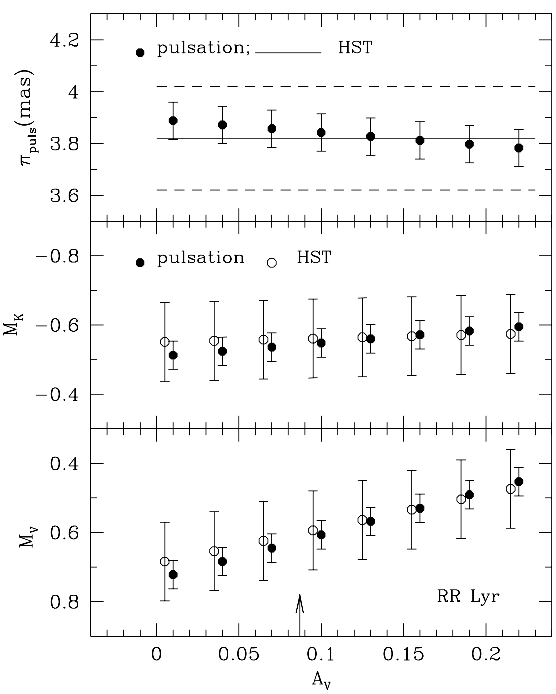

Figure 2. Top panel: predicted pulsation parallax for RR Lyr (filled circles) as a function of the visual extinction. Solid and dashed lines show the HST direct measurement and the relative uncertainty (B02), respectively. . Middle panel: comparison between absolute K-magnitudes of RR Lyr based on HST trigonometric parallax (open circles) and on the pulsation approach (filled circles), as a function of the visual extinction. For clarity sake, HST data are shifted by -0.005 along the x-axis. Bottom panel: the same, but for the absolute visual magnitude. The arrow marks the weighted mean of current extinction corrections

with previous measurements (see B02), data plotted in Fig. 2 show that a sound empirical determination of its absolute magnitude is still hampered by the intrinsic uncertainty on the HST measurement, *even if the interstellar extinction to RR Lyr would be firmly known.*

According to B02, the mean extinction correction to RR Lyr, based on photometric measurements of the astrometric reference stars located close to this variable, is $< A_V >= 0.07 \pm 0.03$ mag. However, the same authors also supply an alternative value $< A_V >= 0.11 \pm 0.10$ mag. Spectroscopic measurements of the diffuse interstellar band at $\lambda = 5780\AA$ ($E(B-V) = -0.01$ mag) and of the Na D-lines ($E(B-V) = 0.12$ mag) carried out by Clementini et al. (1995) do not allow us to settle the problem of the reddening towards RR Lyr. On the other hand, empirical relations connecting the intrinsic color of RR Lyrae stars to the B-band amplitude, the period, and the metallicity (Piersimoni, Bono, & Ripepi 2002) or to the Fourier amplitudes of the V-band light curve (Kovacs & Walker 2001), together with data listed in Table 5, supply a mean reddening of $E(B-V) = 0.029 \pm 0.005$ mag. Finally, we note that the color excess given by Burstein & Heiles (1978) and by Blanco (1992) is $E(B-V) = 0.03 \pm 0.02$ mag, while F98 adopts $E(B-V) = 0.06 \pm 0.03$ mag. As a whole, the weighted mean of extinction corrections available in the literature gives $< A_V >= 0.087 \pm 0.034$ mag (see arrow in Fig. 2).

Table 4. The case of RR Lyr itself. For each adopted extinction correction A_V, column 2 gives the luminosity (in solar units) derived using the equation (7). This luminosity is used in equation (3) to evaluate $M_K(3)$, and in turn to estimate together with $K = 6.54$ mag the intrinsic distance modulus, μ_0, and the "pulsational" parallax π_{puls}. The last two columns give the absolute visual magnitude, based on $V = 7.784$ mag as well as on the pulsational and on the HST absolute parallax (π_{abs}=3.82±0.20 mas), respectively.

A_V (mag)	log$L$(7) ±0.024	$M_K(3)$ (mag) ± 0.041	μ_0 (mag) ± 0.041	π_{puls} (mas) ± 0.072	$M_V^{\pi_{puls}}$ (mag) ± 0.041	M_V^{HST} (mag) ± 0.114
0.01	1.648	−0.513	7.052	3.888	0.722	0.684
0.04	1.664	−0.524	7.060	3.872	0.684	0.654
0.07	1.680	−0.536	7.069	3.857	0.645	0.624
0.10	1.696	−0.548	7.077	3.842	0.607	0.594
0.13	1.712	−0.560	7.086	3.827	0.568	0.564
0.16	1.728	−0.572	7.094	3.812	0.530	0.534
0.19	1.744	−0.583	7.103	3.797	0.491	0.504
0.22	1.760	−0.595	7.111	3.783	0.453	0.474

Table 5. Empirical data for RR Lyr based on the photometry presented by Hardie (1955). The Fourier amplitudes A_1, A_2 and A_3 were estimated by fitting the V-band light curve by means of a cosine series with 7 components.

P (d)	0.5668
(V) (mag)	7.784
(B-V) (mag)	0.371
A_B (mag)	1.154
A_V (mag)	0.892
A_1	0.3014
A_2	0.1493
A_3	0.0920

By adopting this mean reddening value, the HST astrometric measurement gives $M_V = 0.607 \pm 0.114$ (intrinsic HST uncertainty) ± 0.034 mag (A_V contribution) $= 0.607 \pm 0.119$ mag. On the other hand, the pulsational approach gives $\log L = 1.689 \pm 0.030$, and thus $M_V = 0.624 \pm 0.041$ (intrinsic uncertainty on the predicted M_K magnitude) ± 0.044 mag (A_V contribution)$= 0.624 \pm 0.060$ mag.

The final uncertainty on the predicted RR Lyr parameters appears still too large to firmly discriminate among current theoretical or empirical constraints on RR Lyrae intrinsic luminosities and absolute visual magnitude. However, the use of the PLZ_K relation over a large number of well-studied RR Lyrae stars, see next section, that cover a wide metallicity range will provide relevant information on both the slope and, provided that no systematic errors are affecting the pulsating models, the zero-point of the M_V(RR)-[Fe/H] relation.

4 FIELD RR LYRAE STARS

Following the procedure discussed in the previous section for RR Lyr itself, the intrinsic luminosity of individual RR Lyrae stars with well-determined K and V magnitudes, as well as metallicity and reddening, can be directly inferred

Table 6. Observed parameters of RR Lyrae stars for which accurate visual and near-infrared light curves are available. The last column lists the sources of the data. The luminosity given in column (7) was derived using equations (7) [*RRab*] and (8) [*RRc*] and is in solar units. This luminosity is used in equation (3) [*RRab*] and (4) [*RRc*] to derive the M_K magnitude. On the basis of apparent and absolute K magnitudes we evaluate the intrinsic distance modulus, and in turn the absolute visual magnitude.

ID	$\log P$	[Fe/H]	$E(B-V)$	V_0 (mag)	K_0 (mag)	$\log L$ ±0.04	M_K (mag) ±0.05	M_V (mag) ±0.07	Ref.[a]
				Fundamentals					
X Ari	−0.1863	−2.43	0.15	9.078	7.894	1.852	−0.919	0.262	2
M92 V1	−0.1532	−2.24	0.02	15.030	13.810	1.851	−0.965	0.255	8
M92 V3	−0.1956	−2.24	0.02	15.070	13.940	1.855	−0.878	0.252	8
SU Dra	−0.1802	−1.80	0.01	9.761	8.654	1.844	−0.847	0.257	4
VY Ser	−0.1462	−1.79	0.03	10.069	8.780	1.774	−0.866	0.423	2
RV Oct	−0.2430	−1.71	0.13	10.554	9.509	1.798	−0.670	0.375	5
RV Phe	−0.2245	−1.69	0.01	11.873	10.716	1.750	−0.672	0.485	6
RR Leo	−0.3445	−1.60	0.05	10.576	9.660	1.743	−0.403	0.513	4
W Tuc	−0.1923	−1.57	0.01	11.444	10.352	1.816	−0.773	0.319	6
TT Lyn	−0.2237	−1.56	0.01	9.833	8.630	1.711	−0.628	0.575	4
TU UMa	−0.2536	−1.51	0.02	9.764	8.656	1.728	−0.572	0.536	4
SS Leo	−0.2030	−1.50	0.01	11.013	9.933	1.804	−0.733	0.347	7
WY Ant	−0.2410	−1.48	0.05	10.710	9.621	1.751	−0.612	0.477	5
RR Cet	−0.2573	−1.45	0.03	9.625	8.524	1.722	−0.552	0.549	4
M5 V8	−0.2626	−1.40	0.02	15.020	13.960	1.735	−0.544	0.516	8
M5 V28	−0.2645	−1.40	0.02	15.040	13.990	1.739	−0.543	0.507	8
RX Eri	−0.2312	−1.33	0.05	9.529	8.358	1.699	−0.575	0.596	4
M4 V2	−0.2711	−1.30	0.37	12.038	10.918	1.679	−0.473	0.647	9
M4 V15	−0.3529	−1.30	0.37	12.080	11.091	1.660	−0.287	0.702	9
M4 V32	−0.2372	−1.30	0.32	11.956	10.787	1.690	−0.552	0.617	9
M4 V33	−0.2112	−1.30	0.30	11.906	10.745	1.726	−0.633	0.528	9
UU Cet	−0.2175	−1.28	0.03	12.008	10.841	1.713	−0.608	0.559	6
SW Dra	−0.2444	−1.12	0.03	10.389	9.326	1.728	−0.542	0.521	2
UU Vir	−0.3228	−0.87	0.01	10.534	9.516	1.637	−0.279	0.739	4
47 Tuc V9	−0.1326	−0.71	0.04	13.550	12.660	1.925	−0.871	0.019	10
TW Her	−0.3984	−0.69	0.05	11.091	10.217	1.615	−0.082	0.792	2
BB Pup	−0.3180	−0.64	0.10	11.838	10.902	1.669	−0.285	0.651	5
W Crt	−0.3850	−0.54	0.05	11.403	10.543	1.625	−0.099	0.761	5
DX Del	−0.3255	−0.39	0.07	9.714	8.676	1.575	−0.168	0.870	3
RS Boo	−0.4233	−0.36	0.02	10.302	9.445	1.564	0.049	0.906	2
AR Per	−0.3710	−0.30	0.32	9.496	8.532	1.556	−0.048	0.916	4
RR Gem	−0.4009	−0.29	0.08	11.113	10.208	1.555	0.017	0.922	4
SW And	−0.3543	−0.24	0.06	9.510	8.508	1.547	−0.069	0.933	3
V445 Oph	−0.4010	−0.19	0.27	10.131	9.151	1.500	0.070	1.050	7
AV Peg	−0.4085	−0.08	0.07	10.274	9.309	1.489	0.108	1.073	4
RR Lyr	−0.2466	−1.39	0.07	7.714	6.532	1.680	−0.536	0.645	[b]
				First Overtones					
TV Boo	−0.5051	−2.44	0.00	10.984	10.200	1.770	−0.393	0.391	4
T Sex	−0.4885	−1.34	0.05	9.886	9.156	1.729	−0.299	0.431	4
DH Peg	−0.5926	−1.24	0.08	9.287	8.587	1.613	0.019	0.719	1

[a] References: 1) Jones et al. 1988a; 2) Jones et al. 1988b; 3) Jones et al. 1992; 4) Liu & Janes 1990a; 5) Skillen et al. 1993; 6) Cacciari et al. 1992; 7) Fernley et al. 1990; 8) Storm et al. 1994a; 9) Liu & Janes 1990b; 10) Storm et al. 1994b.
[b] See text for more details.

from the measured $V-K$ color, without any *a priori* assumption related with the predicted luminosity of horizontal branch models. Once the individual luminosity of *RRab* variables is derived from equation (7) (equation (8) for *RRc* variables, the predicted PLZ_K relations [see equations (3) and (4)] provide the absolute near-infrared magnitude. This supplies the intrinsic distance modulus, and in turn the absolute visual magnitude. This means that we can investigate both the M_K-[Fe/H]-log$P$ and the M_V-[FeH] relations.

We searched for RR Lyrae stars for which accurate V and K light curves, as well as reliable evaluations of both reddening and metallicity, are available in the literature. A total of 39 stars were found, with 30 field and 9 GGC members. Metal abundances have been taken from the homogeneous compilation by Fernley et al. (1998a). The sample is listed in Table 6 together with key parameters such as the luminosity estimated according to equation (7) or (8), the M_K provided by equation (3) or (4), and the ensuing M_V magni-

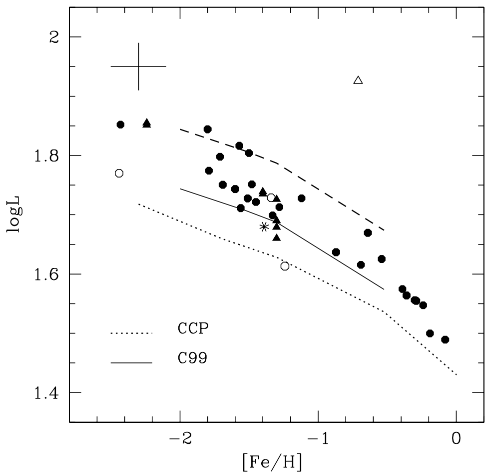

Figure 3. Estimated individual luminosities of RR Lyrae stars in our sample versus metal abundance (see Table 6). Filled and open circles show field *RRab* and *RRc* variables, while filled triangles indicate cluster *RRab* variables. The asterisk and the open triangle show RR Lyr itself and the peculiar RR Lyrae V49 in 47 Tuc, respectively. The error bars display current uncertainties in metal abundance and in luminosity. Dotted and solid lines refer to old and recent ZAHB models, respectively, while the dashed line depicts the exhaustion of the central He-burning phase. See text for more details.

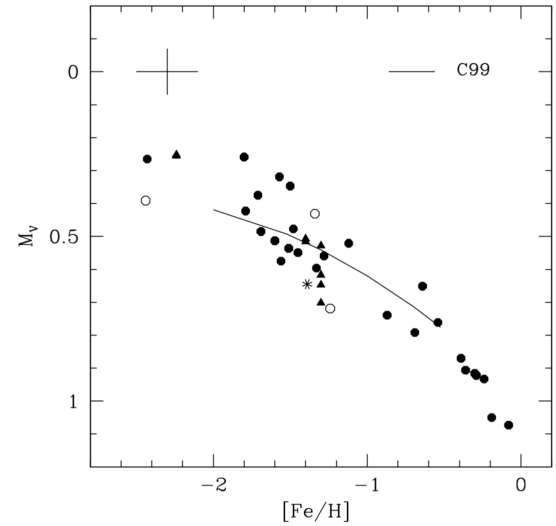

Figure 4. Pulsational absolute visual magnitudes of RR Lyrae variables in our sample as a function of metal content. The symbols are the same as in Fig. 3.

tudes. The errors associated with luminosity, near-infrared, and visual absolute magnitudes account for the intrinsic dispersion of the predicted relations, as well as for an average uncertainty of ±0.20 dex on the metallicity and of ±0.03 mag on A_V.

Figure 3 shows the derived "pulsational" luminosity of RR Lyrae stars in our sample as a function of the metal content. Filled and open circles display field *RRab* and *RRc*, respectively. *RRab* stars in GGCs (M4, M5, and M92) are plotted as filled triangles, while RR Lyr itself with an asterisk. The open triangle marks the position of the RR Lyrae V9 in 47 Tuc which was neglected, since it is peculiar (Storm et al. 1994b). Together with individual RR Lyrae stars, Fig. 3 also shows the predicted $\log L_{RR}$−[Fe/H] relation according to the old ZAHB models by Castellani, Chieffi, & Pulone (1989, 1991, hereinafter CCP, dotted line) as well as the improved computations presented by Cassisi et al. (1998, 1999, hereinafter C99, solid line). The dashed line displays the locus of central He exhaustion according to C99.

As a whole, pulsational luminosities appear in reasonable agreement with the result of recent evolutionary computations based on improved input physics. In fact, with the exception of V9 in 47 Tuc, they show a dispersion in luminosity of the order of 0.10 dex (dashed line) above the theoretical ZAHB, in agreement with the predictions of synthetic horizontal branch simulations (see. e.g. Caputo et al. 1993; Demarque et al. 2000). Thus it seems that recent evolutionary computations and pulsational constraints are supporting each other.

When passing to magnitudes, one has to recall that stellar evolution predicts the luminosity and, therefore, the magnitude of "static" stellar structures, whereas the present pulsational scenario refers to magnitude-weighted mean magnitudes. As discussed by Bono, Caputo & Stellingwerf (1995), variables that present large luminosity amplitudes are expected to have mean visual magnitudes up to ∼ 0.15 mag fainter than the equivalent static values. Bearing in mind this *caveat*, Fig. 4 shows the predicted (magnitude-weighted) absolute visual magnitudes of the variables in our sample as a function of metallicity. The comparison with the ZAHB theoretical predictions by Cassisi et al. (1998, 1999) is now less meaningful and only discloses the compatibility of pulsational and evolutionary scenarios, perhaps suggesting that the evolutionary models are somehow too bright with respect to pulsational magnitudes. However, a more detailed investigation concerning the comparison with current evolutionary predictions requires reliable amplitude corrections to the observed mean magnitudes to obtain the equivalent static values.

It is worth noting that a least-squares solution through the data plotted in Fig. 4 suggests that the M_V-[Fe/H] relation becomes steeper and steeper when moving from metal-poor to metal-rich variables (see Table 7). This result is in agreement with the change in the slope at [Fe/H]∼ −1.5 originally predicted by CCP and further supported by synthetic HB simulations (Caputo et al. 1993; Demarque et al. 2000), by RR Lyrae stars in the field (MacNamara 1999), in

Table 7. The slope of the M_V(RR)-[Fe/H] relation as a function of the adopted metallicity range.

[Fe/H] range	slope
−2.4/−1.8	≈ 0
−2.4/−1.7	0.080±0.100
−2.4/−1.6	0.190±0.090
−2.4/−1.5	0.192±0.079
−2.4/−1.4	0.221±0.060
−2.4/−1.3	0.282±0.053
−2.4/−1.0	0.284±0.050
−2.4/−0.6	0.283±0.039
−2.4/−0.3	0.303±0.022

GGCs (Caputo et al. 2000), and in ω Centauri (Rey et al. 2000).

5 PLZ_K VERSUS FOBE AND BW ABSOLUTE MAGNITUDES

Figure 5 shows the comparison between the absolute visual magnitudes of field RR Lyrae stars estimated using the predicted PLZ_K relation (circles, see column (9) in Table 6) and the averaged absolute visual magnitudes of RR Lyrae stars in GGCs (triangles) derived by Caputo et al. (2000) according to the "First Overtone Blue Edge" (FOBE) method. The agreement between these two independent approaches is excellent, thus supporting the consistency of the pulsational scenario. At the same time,this result indicates that field and cluster RR Lyrae stars should have experienced quite similar evolutionary histories.

As a whole, linear interpolation through the data plotted in Fig. 5 gives the following analytical relations (solid lines):

$$M_V = 0.718(\pm 0.072) + 0.177(\pm 0.069)[Fe/H] \quad (9)$$

for RR Lyrae stars with [Fe/H]≤ -1.6, and

$$M_V = 1.038(\pm 0.077) + 0.359(\pm 0.027)[Fe/H] \quad (10)$$

for RR Lyrae stars with [Fe/H]> -1.6.

Let us now compare our results with current distance determinations based on the BW method. To perform a robust comparison we adopted the absolute visual magnitudes for field RR Lyrae stars derived by Fernley (1994). The key feature of this sample is that the empirical data available in the literature were reduced by the previous author to the same NIR BW method. At the same time, for these objects the metal abundances were also estimated by adopting a homogeneous metallicity scale (Fernley et al. 1998a). We ended up with a sample of 27 RR Lyrae stars. The nine GGC variables were not included, since they appear to be affected by larger intrinsic errors and also because they have not been reduced to the same BW method yet. Figure 6 shows the comparison between these NIR BW absolute magnitudes (filled triangles) and the absolute visual magnitudes for the same objects, as based on the PLZ_K relation (circles). Data plotted in this figure display quite clearly that for RR Lyrae stars more metal-poor than [Fe/H] ~ -1.5, the discrepancy

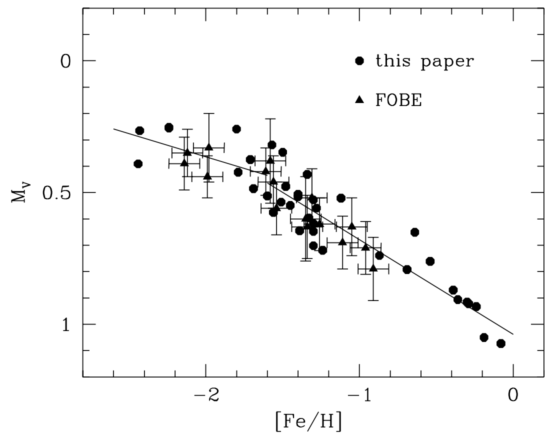

Figure 5. Comparison between absolute visual magnitudes of RR Lyrae variables in our sample evaluated using the current pulsation approach (circles) and the results of First Overtone Blue Edge (FOBE) method (triangles) to globular cluster variables (Caputo et al. 2000). The solid lines are the relations described in the text.

is on average of the order of 0.2 mag, in the sense that BW magnitudes are fainter than those provided by current pulsational approach. The discrepancy decreases toward higher metal contents and vanishes in the metal-rich tail. This indicates that the metallicity dependence given by the BW method is milder than predicted by pulsation models.

Note that a new calibration of the BW method has been provided by Fernley et al. (1998b), but the difference over the entire metallicity range is negligible when compared with Fernley (1994). On the contrary, the preliminary analysis provided by Cacciari et al. (2000) of the assumptions currently adopted in the BW method disclosed quite different results. In particular, by using available photometric data together with a large set of new atmosphere models, they found that RR Cet ([Fe/H]=-1.45) is ≈ 0.12 mag brighter than previously estimated, whereas no significant correction was required for SW And ([Fe/H]=-0.24). As a result, these new absolute magnitudes (open squares in Fig. 6), once confirmed, appear fully consistent with the M_V-[Fe/H] relation based on pulsation models [equation (10)]. This suggests that the slope of this relation, at least in the metallicity range $-1.5 \leq$[Fe/H]≤ -0.4, is steeper than currently adopted (see also McNamara 1999).

Following a referee's suggestion, one may investigate whether the discrepancy at [Fe/H] < -1.5 between pulsational absolute visual magnitudes and BW magnitudes might be settled by changing the mass value in equations (1) and (5). We found that to remove the difference of

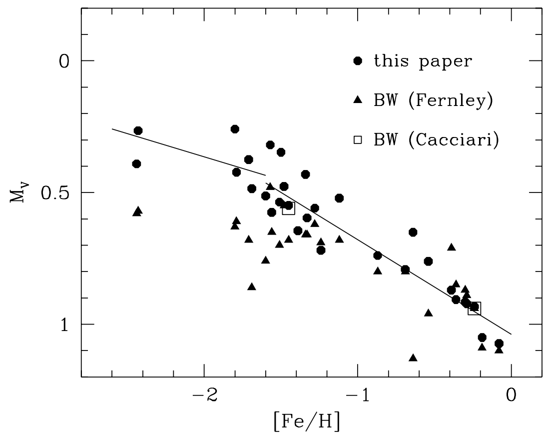

Figure 6. Comparison between the absolute visual magnitudes estimated using the current pulsation approach and the independent evaluations derived by Fernley (1994) using the NIR BW method (triangles). The two open squares mark the position of RR Cet ([Fe/H]=−1.45) and SW And ([Fe/H]=−0.24) according to the new calibration of the BW method provided by Cacciari et al. (2000). The solid lines are the relations described in the text.

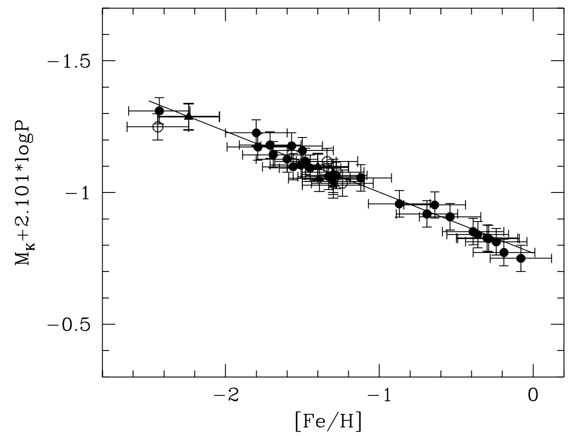

Figure 7. Absolute K-magnitudes of RR Lyrae in our sample estimated using the pulsation method as a function of metal content. Data were projected onto a two dimensional plane. The solid line shows the projection onto the same plane of equation (11). The symbols are the same as in Fig. 3.

$\sim$ 0.25 mag between BW and pulsational magnitudes at [Fe/H]=−1.8, one should account for a decrease in mass of the order of $\sim$ 23%, thus predicting a quite unrealistic low-mass, $0.55M_\odot$, for these metal-poor ($Z \sim 0.0003$) horizontal branch stellar structures.

As for K-magnitudes, Fig. 7 shows that the absolute K-magnitudes estimated using the pulsational approach do obey a very tight relation with the metal content, once the dependence on period is removed. Owing to the small number (3) of *RRc* variables, their periods have been fundamentalized, (logP_F=logP_{FO}+0.127), thus yielding for the entire sample the following relation:

$$M_K^F = -0.770(\pm0.044) - 2.101 \log P + 0.231(\pm0.012)[\mathrm{Fe/H}] \quad (11)$$

The small errors affecting the coefficients of this relation further

strengthen the evidence that near-infrared observations could supply accurate distance determinations.

To investigate in more detail the discrepancy between the current approach and the BW method, we compare the pulsational absolute K magnitudes with the absolute K magnitude obtained using the BW true distance modulus and the dereddened apparent K magnitudes listed in column (6) of Table 6. Figure 8 shows the comparison between pulsational M_K values (circles) and results based on the BW method (triangles). Once again, the BW magnitudes at [Fe/H]$\leq -$ 1.5 are fainter than predicted by the pulsational approach, leading to a milder dependence on [Fe/H] with respect to the PLZ_K predictions (solid line). However, the absolute K-magnitudes derived by Cacciari et al. (2000) for SW And and RR Cet (open squares) appear once again in good agreement with our pulsational approach.

6 SUMMARY AND FINAL REMARKS

Data plotted in Fig. 6 and Fig. 8 clearly show that the pulsational approach provides absolute magnitudes for RR Lyrae stars that are significantly brighter than suggested by the BW method. This difference is more evident for metal-poor ([Fe/H]$\leq -$ 1.5) RR Lyrae variables. The recent revision provided by Cacciari et al. (2000) for RR Cet ([Fe/H]=−1.45) and SW And ([Fe/H]=−0.24) agrees quite well with the pulsational results. This suggests that both the zero-point and the slope of the M_V-[Fe/H] and of the M_K-[Fe/H]-log$P$ relation based on the old BW calibration need to be revisited.

We have also found that both V and K absolute magnitudes of RR Lyr itself, estimated via the pulsational approach, are in good agreement with the trigonometric parallax recently measured by HST (see Fig. 2). This suggests that at least for the metallicity of RR Lyr ([Fe/H]=−1.39), the pulsational approach is consistent with direct distance determinations.

As far as the metal-dependence is concerned, we finally

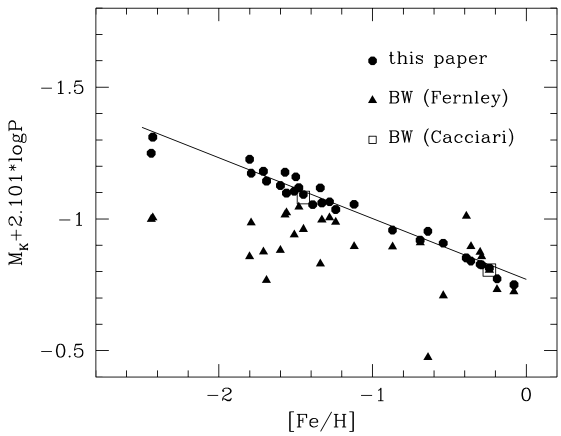

Figure 8. Comparison between the absolute K-magnitudes based on the pulsation approach and the BW method. The latter were derived using the true distance moduli given by Fernley (1994) and the K_0-magnitudes listed in Table 6. The solid line refers to equation (11).

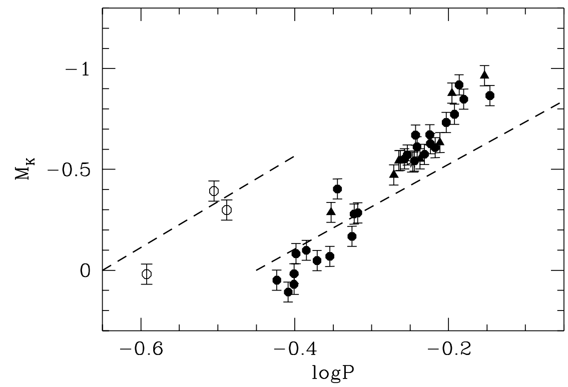

Figure 9. Absolute K-magnitudes for the current sample of well-observed field and cluster RR Lyrae stars. Symbols are the same as in Fig. 3. The dashed lines depict the predicted slope at constant luminosity and metal content.

note that the effect of metallicity on equation (11) can be revealed by a simple cross-check between predicted K magnitudes and periods. It has been already shown (see Paper I) that K measurements for RR Lyrae stars in M3, namely for variables at quite constant luminosity and metal content, closely follow the pulsational constraints on the period dependence. On the contrary, data plotted in Fig. 9 show that for the current sample of cluster and field RR Lyrae stars the M_K-log$P$ relation is significantly steeper than predicted under the above quoted assumptions (dashed lines). This is the expected aftermath of the fact that metal-poor variables, which have longer periods (see data in Table 6), have also brighter bolometric luminosities (see Fig. 3) than metal-rich ones.

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