Extended Skyrme Interaction in the Spin Channel

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Most of the Skyrme interactions are known to predict spin or isospin instabilities beyond the saturation density of nuclear matter which contradict predictions based on realistic interactions. A modification of the standard Skyrme interaction is proposed so that the ferromagnetic instability is removed. The new terms are density dependent and modify only the spin p-h interaction in the case of spin-saturated system. We have shown that these new terms change the total binding energy of odd-nuclei by only few tenths of keV. From the analysis of the spin instabilities of nuclear matter, restrictions on the parameters governing the spin-density dependent terms are evaluated. We conclude that with the extended Skyrme interaction, the Landau parameters G_0 and G_0' could be tuned with a large flexibility without changing the ground-state properties in nuclei and in nuclear matter.

Despite many theoretical and experimental investigations, the spin and the spin-isospin channels in either the ground and the excited states of nuclei are still widely open for future study.[1)–5)] It is indeed difficult to probe the spin and the spin-isospin channels of nuclear interaction since the ground states of nuclei are non-spin polarized in the case of even-even nuclei and at most polarized by the last unpaired nucleons in odd nuclei. However, as widely used Skyrme interaction have unrealistic values of G_0', it is undeniable that adding more flexibility to the spin-isospin part of Skyrme forces is useful.[6)]

In Ref. 7), new spin-density dependent terms have been introduced on top of standard Skyrme forces in order to remove the ferromagnetic instability associated with all Skyrme parameterizations:[9)]

$$V^{s,st}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{6}t_3^s(1+x_3^sP_\sigma)[\rho_s(\boldsymbol{R})]^{\gamma_s}\delta(\boldsymbol{r}) + \frac{1}{6}t_3^{st}(1+x_3^{st}P_\sigma)[\rho_{st}(\boldsymbol{R})]^{\gamma_{st}}\delta(\boldsymbol{r}), \quad (1)$$

where $P_\sigma = (1+\sigma_1\cdot\sigma_2)/2$ is the spin-exchange operator, $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\boldsymbol{R} = (\boldsymbol{r}_1+\boldsymbol{r}_2)/2$. In Eq. (1), the spin-density $\rho_s \equiv \rho_\uparrow - \rho_\downarrow$ and the spin-isospin-density $\rho_{st} \equiv \rho_{n\uparrow} - \rho_{n\downarrow} - \rho_{p\uparrow} + \rho_{p\downarrow}$ have been introduced. Spin symmetry is satisfied if the power of the density-dependent terms γ_s and γ_{st} is even.

The new terms retain the simplicity and the good properties of Skyrme interactions for nuclear matter and the ground states of even-even nuclei. In this work, we first study the properties of nuclear matter. The additional terms (1) modify the standard Skyrme energy density functional[6),7)] The additional contributions to the mean field are zero in even-even nuclei. Since most of the Skyrme interactions are adjusted on (few) even-even nuclei, it is thus possible to add for these interactions the new terms (1) in a perturbative manner. The new four parameters t_3^s, x_3^s, t_3^{st} and x_3^{st} in Eq. (1) have been adjusted in Ref. 7) in order to reproduce the Landau parameters extracted from a G-matrix calculation in uniform matter, while $\gamma_s = \gamma_{st} = 2$ is imposed by spin symmetry.

From the analysis of the Landau parameters the stability around spin-symmetric matter requires that $x_3^s < 1$ (see Eq. (11) of Ref. 7)), while the ground state is absolute for non-polarised matter if $-1 \lesssim x_3$. As a conclusion, the parameter x_3^s inside the range shall be adjusted inside the range, $-1 \lesssim x_3^s < 1$.

The case of pure neutron matter is somehow very peculiar. The correction (1) is zero for spin polarised matter. This property is related to the anti-symmetrization of the interacting nucleons. Indeed, in fully polarized neutron matter, the quantum numbers for spin and isospin are $S = 1$ and $T = 1$ while the new spin-density dependent interaction (1) acts in the $L = 0$ channel. The new spin-density dependent interaction have thus no effect at all in the purely spin-polarized neutron matter.

The new terms are non-zero in odd-systems. Therefore, we also study the effects of these new terms on the ground state properties of odd nuclei, in particular the total binding energy and the density distribution. To provide an approximate maximal estimate of the effects while keeping our model simple, we perform HF calculations with the following approximation to treat odd nuclei. We use the equal filling approximation, so that the time-reversal symmetry is not broken, but with an additional ansatz. As within this scheme the spin-densities would be by definition equal to zero, when constructing the spin-densities with the wave function of the odd nucleon, we assume that the spin-up state is completely filled while the spin-down state is empty between the two possible spin orientations (or, equivalently, the opposite). We call this procedure for the construction of spin-densities the *one-spin polarized approximation* (OSPA). The OSPA gives an upper value of the contribution of the new terms.

We have made several calculations for the ground-state energies in ^{41,42,49,50}Ca that are summarized in Tables I and II. The partial contributions to the ground-state energy are written $E_{\rm MF} \equiv \int d^3r\ (\mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm sg})$ for the mean field, $E_{\rm so}$ for the spin-orbit, $E_{\rm Coul}$ for the Coulomb, and $E_{\rm kin}$ for the kinteic contribution, using the notations of Ref. 10). In Table I the total energy, the mean field, the spin-orbit, the Coulomb and the kinetic contributions to the total energy and the single-particle energy (for the neutron states $1f_{7/2}$ or $2p_{3/2}$) are provided for the odd nuclei ^{41}Ca and ^{49}Ca and for the nearest even-even nuclei ^{42}Ca and ^{50}Ca. The calculation for the odd nuclei are performed either with or without the spin-density dependent interaction (1). The difference between the total energy, and its contributions, without and with the corrections ΔE is shown to be less than 50 keV in both odd nuclei ^{41}Ca and ^{49}Ca. It is interesting to notice that the kinetic energy contributes to reduce the impact of the spin-dependent interaction (1). The spin-orbit and the Coulomb energies are very weakly affected by the spin-density terms.

Table I. Total energy, mean field, spin-orbit, Coulomb and kinetic contributions to the total energy (third column) and single-particle energy of the neutron state $1f_{7/2}$ for $^{41-42}$Ca and $2p_{3/2}$ for $^{49-50}$Ca calculated, for the nearest even nuclei and for the odd nuclei, without/with the spin-dependent terms (1) in the mean field. ΔE is the difference of energy with and without the spin-dependent terms (1).

Nucleus	$E_{\rm TOT}$ (MeV)	$E_{\rm MF}$ (MeV)	$E_{\rm so}$ (MeV)	$E_{\rm Coul}$ (MeV)	$E_{\rm kin}$ (MeV)	s.p. energy (MeV)
^{42}Ca	−362.591	−1111.434	−9.173	72.023	685.993	−9.66
^{41}Ca	−352.942	−1081.395	−5.259	72.116	661.596	−9.64
^{41}Ca with (1)	−352.918	−1081.359	−5.259	72.115	661.584	−9.64
ΔE	0.024	0.036	0.000	−0.001	−0.012	
^{50}Ca	−429.654	−1326.381	−33.958	70.905	859.779	−5.84
^{49}Ca	−423.876	−1305.865	−33.639	71.105	844.523	−5.70
^{49}Ca with (1)	−423.825	−1305.754	−33.634	71.102	844.461	−5.70
ΔE	0.051	0.111	0.005	−0.003	−0.062	

Table II. Total and separate contributions to the energy from the time-odd (spin symmetry breaking) terms of the SLy5 Skyrme interaction.

Nucleus	$E^{\rm odd}_{\rm TOT}$ (MeV)	$E^{\rm odd}_0$ (MeV)	$E^{\rm odd}_{\rm eff+fin+sg}$ (MeV)	$E^{\rm odd}_3$ (MeV)
^{41}Ca	0.329	0.196	−0.007	0.140
^{49}Ca	0.151	0.187	−0.176	0.140

In Table II the total and separate contributions coming from the time-odd terms of the Skyrme interaction (see Eqs. (A.1)–(A.5) of Ref. 6)) are provided, in the case of the odd nuclei ^{41}Ca and ^{49}Ca: these contributions are calculated perturbatively within the OSPA. These terms are classified according to the standard notations and labeled as $E^{\rm odd}_0$, $E^{\rm odd}_{\rm eff+fin+sg}$ and $E^{\rm odd}_3$ which are the contributions from the central, the momentum dependent and the density dependent terms, respectively. The corrections $E^{\rm odd}_0$ and $E^{\rm odd}_3$ give a dominant and repulsive contribution which increases the total energy while the correction $E^{\rm odd}_{\rm eff+fin+sg}$ is smaller and attractive. The total correction remains quite small, that is, of the order of 0.15–0.3 MeV for both nuclei. Note that the sign of these corrections could change from one Skyrme interaction to another, but such corrections in Table II remain larger than $\Delta E_{\rm TOT}$ in Table I. From the quantitative comparison shown in Tables I and II, we can infer, as expected, that the new spin-density dependent terms (1) modify the ground state energies of odd nuclei much smaller than those coming from the time-odd terms of the standard Skyrme interaction.

In this work we have carefully analyzed the ground state properties of finite nuclei and infinite matter obtained by the extended Skyrme interactions with the spin-density terms proposed in Ref. 7). In finite even nuclei, the new spin-density interactions are simply zero and from the OSPA we have shown that these terms has only negligible contributions to the ground-state of odd nuclei. Thus, it has been shown that by using the extended Skyrme interactions,[7)] the Landau parameters G_0 and G'_0 could be tuned to realistic values without altering the ground-state properties in odd nuclei.

Acknowledgements

S.G. acknowledges financial support from FNRS. The work is partially supported by COMPSTAR, an ESF Research Networking Programme, the ANR NExEN contract and the Japanese Ministry of Education, Culture, Sports, Science and Technology by a Grant-in-Aid for Scientific Research under the program numbers (C) 20540277.

References

1) F. Osterfeld, Rev. Mod. Phys. **64** (1992), 491.
2) E. T. Suzuki and H. Sakai, Phys. Lett. B **455** (1999), 25.
M. Ichimura, H. Sakai and T. Wakasa, Prog. Part. Nucl. Phys. **56** (2006), 446.
T. Wakasa, M. Ichimura and H. Sakai, Phys. Rev. C **72** (2005), 067303.
3) I. N. Borzov, S. V. Tolokonnikov and S. A. Fayans, Sov. J. Nucl. Phys. **40** (1984), 732.
4) I. N. Borzov, Nucl. Phys. A **777** (2006), 645.
5) S. Fracasso and G. Colò, Phys. Rev. C **76** (2007), 044307.
6) J. Margueron, S. Goriely, M. Grasso, G. Colò and H. Sagawa, J. of Phys. G **36** (2009), 125103.
7) J. Margueron and H. Sagawa, J. of Phys. G **36** (2009), 125102.
8) J. Margueron, J. Navarro and N. V. Giai, Phys. Rev. C **66** (2002), 014303.
9) J. Margueron, J. Navarro and N. V. Giai, Phys. Rev. C **66** (2002), 014303.
10) E. Chabanat et al., Nucl. Phys. A **627** (1998), 710; Nucl. Phys. A **635** (1998), 231; Nucl. Phys. A **643** (1998), 441.