G. Parisi: A DUALITY SUM RULE FOR DEEP INELASTIC SCATTERING.

A Duality Sum Rule for Deep Inelastic Scattering.

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Recently, Bloom and Gilman (1) have proposed to extend duality to deep inelastic scattering.

Their hypothesis is that $F_2(\omega) = \lim_{s \to 0} W_s(s, q^2)$ can be decomposed into two terms, resonances and background; the background is only $I = 0$ in the $t$-channel and the resonance contribution is not zero.

In this letter we want to show that from the assumptions of duality, $SU_3$ symmetry, absence of $s$-channel and $t$-channel exotic contributions and validity of Fubini-Dashen-Gell-Mann (2) sum rule, we are able to derive a sum rule (3) for the difference between proton and neutron structure functions

$$
\int_0^1 [F_{2p}(\omega) - F_{2n}(\omega)] \frac{d\omega}{\omega} = \frac{1}{3}.
$$

This sum rule is nearly satisfied: the experimental value for the left hand side is 0.19 with an error of 40% (4). Of course, we do not expect our sum rule to be exactly true in the real world: it may have a 20% error due to $SU_3$ breaking. This sum rule was also obtained by Landshoff and Polkinghorne (5) in the framework of the quark-parton model.

The derivation of eq. (1) is very simple. We denote by $F_{2p}^{th}(\omega)$ the structure functions relative to $\langle B|J^+(\omega)F_1(0)|B\rangle$, where $B$ is or the one-proton or the one-neutron state and $J^+_s(\omega)$ are the eight $SU_3$ currents.

Duality allows us to decompose $F_{2p}^{th}(\omega)$ into resonances and background, and the background is only a $t$-channel $SU_3$ singlet.

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Under the hypothesis of absence of exotic t-channel exchange \((10, \overline{10}, 27)\), we can write the resonance contribution in the following way:

\begin{equation}
\mathcal{B}^{(1)}(\omega) = C(\omega) \delta_{123} + D(\omega) \delta_{14} \pm d(\omega) \delta_{13},
\end{equation}

where we must take the sign \(+\) for the proton and \(-\) for the neutron.

The absence of \(s\)-channel exotic resonances \((\overline{10}, 27)\) fixes

\begin{equation}
d(\omega) + f(\omega) = 0.
\end{equation}

From the Fubini–Dasen–Gell-Mann \((^2)\) sum rules

\begin{equation}
\int_0^1 \frac{d\omega}{\omega} = \frac{1}{4},
\end{equation}

follows that

\begin{equation}
\int_0^1 \left[ F_{12}^{\pm}(\omega) - F_{13}^{\pm}(\omega) \right] \frac{d\omega}{\omega} = -\frac{4}{3} \int_0^1 \frac{d\omega}{\omega} d\omega = \frac{1}{3}.
\end{equation}

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