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Research Article

Capacity of Wireless Ad Hoc Networks with Opportunistic Collaborative Communications

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Optimal multihop routing in ad hoc networks requires the exchange of control messages at the MAC and network layer in order to set up the (centralized) optimization problem. Distributed opportunistic space-time collaboration (OST) is a valid alternative that avoids this drawback by enabling opportunistic cooperation with the source at the physical layer. In this paper, the performance of OST is investigated. It is shown analytically that opportunistic collaboration outperforms (centralized) optimal multihop in case spatial reuse (i.e., the simultaneous transmission of more than one data stream) is not allowed by the transmission protocol. Conversely, in case spatial reuse is possible, the relative performance between the two protocols has to be studied case by case in terms of the corresponding capacity regions, given the topology and the physical parameters of network at hand. Simulation results confirm that opportunistic collaborative communication is a promising paradigm for wireless ad hoc networks that deserves further investigation.

1. INTRODUCTION

The emergence of novel wireless services, such as mesh-based wireless LANs and sensor networks, is causing a shift of the interest of the communications community from infrastructure-based wireless networks to ad hoc wireless networks. While the theory of infrastructure-based wireless networks is by now fairly well developed, a complete information-theoretic characterization of ad hoc wireless networks is still far from being realized, even in the simplest cases of relay channels or interference channels.

In recent years, landmark works that address this knowledge gap have been published, by relying mostly on asymptotics or simplified assumptions. In [1], the scaling law of the transport capacity (measured in bps per meter) versus the number of nodes that was derived under the assumption of a static network with multihop (MH) and point-to-point coding. A different approach was pursued in [2], where a general framework for the computation of the capacity region of wireless networks under given transmission protocols was proposed. The protocols considered in [2] included single/multihop transmission with or without spatial reuse, power control, and successive interference cancellation. Overall, the works reported above, and the literature stemmed from these references, concentrate on MH and fail to account for one of the most promising wireless transmission technologies, namely, cooperation (see, e.g., [3]). An attempt in this direction was made in [4] where the capacity region of an ad hoc network with single-relay amplify-and-Forward (AF) transmission was studied.

A major observation in interpreting the capacity region of [2] with MH is that in order to achieve the points on the boundary of the region, optimal time-division scheduling among the basic transmission schemes has to be employed. This requires coordination among the nodes on a global level, which in turn implies the need for the exchange of overhead information at higher layers of the protocol stack (MAC and network) [5]. As a valid alternative, this paper studies the performance of an ad hoc network under the collaborative space-time coding scheme investigated in [6] (The words "cooperation" and "collaboration" will be used interchangeably throughout the paper.). According to this strategy, originally presented in the context of single-link relayed transmission, cooperation with a transmitting source occurs opportunistically, that is, whenever an idle node is able to decode the transmitted signal before the intended destination.

We refer to this scheme as *opportunistic space-time collaboration* (OST).[1] In [6] an achievable rate for OST was derived under the assumption that channel state information is only available at the receiving side of each wireless link.

The main contribution of this paper is twofold. (1) It is shown analytically that the (distributed) OST scheme outperforms (centralized) optimal MH transmission in a scenario where no spatial reuse is allowed (i.e., multiple concurrent transmissions are not allowed). In other words, the capacity region achievable by OST is larger than that obtained by MH. This conclusion is obtained by exploiting the results in [6] and by casting the optimal MH problem of [2] in a suitable framework. (2) If spatial reuse is employed, the increased interference caused by the opportunistic transmission of idle nodes in OST can be deleterious to concurrent transmissions, and optimized MH transmission may be advantageous in some cases. Simulation results show that the relative performance of OST and MH for spatial reuse should be studied case by case, given the topology and the physical parameters of network at hand.

Notation

Lowercase (uppercase) bold denotes column vector (matrix); v_i denotes the ith element of the $N \times 1$ vector $\mathbf{v}$ $(i = 1, \ldots, N)$; A_{nm} is the (n, m)th element of the $N \times M$ matrix $\mathbf{A}$ $(n = 1, \ldots, N,\ m = 1, \ldots, M)$.

2. SYSTEM MODEL

Consider an ad hoc network with n single-antenna nodes, collected in the set $\mathcal{N} = [1, \ldots, n]$. Each node may want to communicate (an infinite backlog of) data to another single node (no multicast is allowed), possibly through MH or collaborative transmission. A node that generates a data stream is referred to as the *source* node for the given data stream, whereas the node to which the data stream is finally intended is called the *destination*. When active, each node transmits with power P [W] and is not able to receive simultaneously (half duplex constraint).

A pair of nodes i and $j \in \mathcal{N}$ is separated by a distance d_{ij} [m]; moreover, the wireless link between the ith and jth nodes is characterized by a (Rayleigh) fading coefficient $h_{ij} \sim \mathcal{CN}(0, 1)$. The overall channel power gain between the two nodes reads

$$G_{ij} = \rho_0 \left(\frac{d_0}{d_{ij}}\right)^{\alpha} \left|h_{ij}\right|^2, \tag{1}$$

where d_0 is a reference distance, α is the path loss exponent and ρ_0 is an appropriate constant setting the signal-to-noise ratio (SNR) at the reference distance. Notice that for reciprocity, $h_{ij} = h_{ji}$ and thus $G_{ij} = G_{ji}$.

Let us denote by $\mathcal{A} \subset \mathcal{N}$ the set of active (transmitting) nodes at a given time instant. In a *collaborative* scenario, possibly more than one node in $\mathcal{A}$ are active transmitting to a given node j. Therefore, the set $\mathcal{A}$ can be partitioned into nonoverlapping subsets $\mathcal{A}_j$, where $\mathcal{A}_j$ denotes the set of nodes cooperating for transmission to j. Notice that transmission from nodes in $\mathcal{A} \backslash \mathcal{A}_j$ causes interference on the reception of node j. As for any collaborative technique that requires the cooperating node to fully decode the signal (e.g., decode and forward (DF) schemes [3]), the nodes in $\mathcal{A}_j$ are assumed to have decoded the signal intended for node j by the considered time instant. Moreover, assuming no channel state information at the transmitter, the signals from different cooperating nodes add incoherently at the receiver and the resulting SINR for reception at node j reads

$$\mathrm{SINR}_j\left(\mathcal{A}_j, \mathcal{A}\right) = \frac{\sum_{k \in \mathcal{A}_j} G_{kj} P}{N_0 B + \sum_{k \in \mathcal{A} \backslash \mathcal{A}_j} G_{kj} P}, \tag{2}$$

where N_0 is the power spectral density of the background noise [W/Hz] and B is the signal bandwidth. Notice that the SINR for collaborative transmission (2) reduces to the standard SINR for a noncollaborative scenario in case only one node is active for transmission to any receiving node j, that is, $\mathcal{A}_j = \{i\}$, $i \in \mathcal{N}$. The channel capacity [bps] on the wireless link between the set of nodes $\mathcal{A}_j$ and j is

$$C_j\left(\mathcal{A}_j, \mathcal{A}\right) = B \cdot \log_2\left(1 + \mathrm{SINR}_j\left(\mathcal{A}_j, \mathcal{A}\right)\right). \tag{3}$$

3. COLLABORATIVE COMMUNICATIONS IN AD HOC NETWORKS: NO SPATIAL REUSE

In this section, performance comparison between (centralized) optimal MH and the (distributed) OST scheme proposed in [6] is presented in terms of achievable rates for ad hoc networks with no spatial reuse (i.e., multiple concurrent transmissions are not allowed). This requires to cast the optimal MH problem into a convenient framework (Section 3.1) and to exploit the results in [6] for the case where any number of nodes can collaborate with the ongoing transmission (Section 3.2).

The discussed performance comparison between optimal MH and the distributed OST scheme aims at showing that the overhead of setting up a centralized optimization procedure could be avoided by cooperative techniques at the physical layer without compromising (or even increasing) the system performance. However, it should be noted that the comparison is not fair from the standpoint of the total transmission power employed by the network. In fact, the total transmission power in the OST scheme is not directly controllable due to lack of channel state information at the transmitters, and may exceed the power spent by optimal MH. In energy-constrained networks, it is then necessary to assess the best solution as a trade-off between the energy needed to make centralized MH optimization feasible and the extra transmission energy required by OST.

[1] Notice that the term *opportunistic* is used here in the same sense of [7], where a practical (uncoded) implementation of OST is investigated.

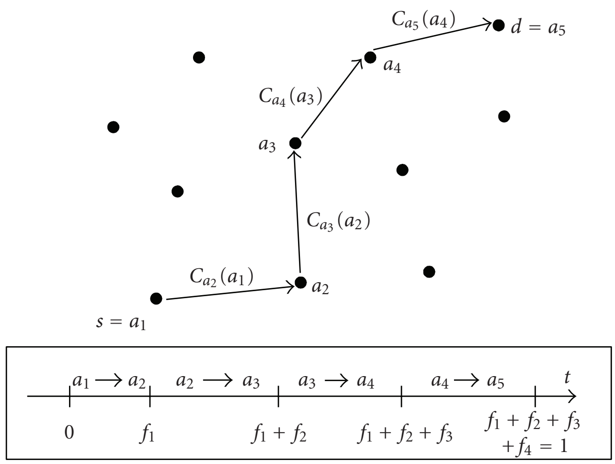

Figure 1: Illustration of an MH route ($M = 3$ hops).

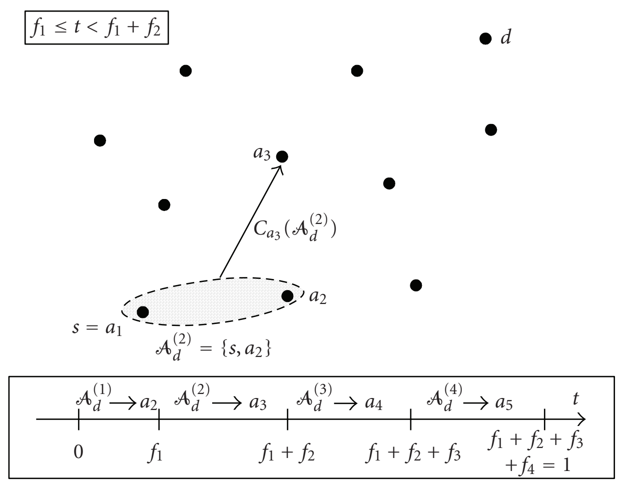

Figure 2: Illustration of the OST scheme ($M = 3$).

3.1. *Optimal multihop transmission*

Consider Figure 1. Source node s generates a data stream intended for node d. Since we are focusing on a case with no spatial reuse, only one source-destination pair is active. According to [2], maximizing the rate R_{sd} between s and d entails centralized optimization of the time schedule among the $\breve{M} = n(n-1)+1$ basic transmission modes allowed by the MH protocol with no spatial reuse. Here, for convenience of analysis, we restate the problem of maximizing R_{sd} in the following equivalent way. Find (i) the sequence of $M+1$ (with $0 \le M \le n-2$) hops, that we denote by the $(M+2)\times 1$ vector $\mathbf{a}$, with $a_1 = s$ and $a_{M+2} = d$; (ii) the $(M+1)\times 1$ optimal scheduling vector $\mathbf{f}$, where f_m refers to the fraction of time devoted for the hop from node a_m to a_{m+1}, such that

$$R_{sd}^{MH} = \max_{\{M,\mathbf{a},\mathbf{f}\}} \left(\min_{m=1,\dots,M+1} f_m C_{a_{m+1}}(a_m) \right) \tag{4}$$

is subject to $\sum_{m=1}^{M+1} f_m = 1$, where we have defined for simplicity of notation $C_j(\mathcal{A}_j) = C_j(\mathcal{A}_j, \mathcal{A}_j)$ (recall (2) and (3)). See Figure 1 for a pictorial view of the problem. From (4), it is clear that the optimal MH route maximizes the bottleneck of the weakest link along the route. Formulation of the optimal MH problem as in (4) allows the performance comparison with the OST scheme, as shown in the next section.

3.2. *Opportunistic space-time cooperation*

Consider again the situation in Figure 1. According to OST, the source node starts the transmission at a given rate R_{sd}, not being informed of whether the signal will arrive to the destination directly or by collaborative transmission. As soon as any node $a_2 \in \mathcal{N}\backslash\mathcal{A}_d^{(1)}$ (where $\mathcal{A}_d^{(1)} = \{s\}$ is the set of collaborating nodes) is able to decode the signal from s, it starts transmitting a cooperating signal (see Figure 2). We denote the (normalized) time instant when successful decoding of the first cooperating node takes place as $0 < f_1 \le 1$:

$$f_1 = \min_{a_2 \in \mathcal{N}\backslash\mathcal{A}_d^{(1)}} \frac{R_{sd}}{C_{a_2}(\mathcal{A}_d^{(1)})}. \tag{5}$$

Node a_2 is able to calculate f_1 since it is assumed to know the channel gain G_{sa_2} (channel state information at the receiving sides), and therefore the capacity $C_{a_2}(s)$. Notice that if there is no node a_2 that has a channel capacity from the source such that $C_{a_2}(\mathcal{A}_d^{(1)}) > R_{sd}$, then we set $a_2 = d$, and no collaboration occurs. Otherwise, the signal transmitted by nodes s and a_2 might be successfully decoded by a third node $a_3 \in \mathcal{N}\backslash\mathcal{A}_d^{(2)}$ ($\mathcal{A}_d^{(2)} = \{s, a_2\}$), as shown in Figure 2. Node a_3 may or may not be equal to d and the time of successful decoding is $0 < f_1 + f_2 \le 1$ with

$$f_2 = \min_{a_3 \in \mathcal{N}\backslash\mathcal{A}_d^{(2)}} \frac{R_{sd} - f_1 C_{a_3}(\mathcal{A}_d^{(1)})}{C_{a_3}(\mathcal{A}_d^{(2)})}. \tag{6}$$

In (6), the numerator is proportional to the number of bits that node a_3 still needs to decode at time f_1; thus, dividing by the capacity $C_{a_3}(\mathcal{A}_d^{(2)})$, we get the additional time that a_3 needs in order to decode the message. At $f_1 + f_2$, the third node starts collaborating and the procedure repeats with

$$f_m = \min_{a_{m+1} \in \mathcal{N}\backslash\mathcal{A}_d^{(m)}} \frac{R_{sd} - \sum_{k=1}^{m-1} f_k C_{a_{m+1}}(\mathcal{A}_d^{(k)})}{C_{a_{m+1}}(\mathcal{A}_d^{(m)})}, \quad m = 1,\dots,M, \tag{7}$$

and $\sum_{m=1}^{M} f_m < 1$.

At the end of the transmission, $0 \le M \le n-2$ nodes cooperate with the source s and thus belong to the set of active nodes $\mathcal{A}_d^{(M+1)}$. The activating order is defined by the $(M+2)\times 1$ vector $\mathbf{a} = [a_1 = s,\ a_2, \dots, a_{M+2} = d]^T$ and the corresponding activating times are in the $(M+1)\times 1$ vector $\mathbf{f}$ ($f_{M+1} = 1 - \sum_{m=1}^{M} f_m$). See Figure 2 for an illustration of the procedure. The rate achievable by this distributed greedy procedure is [6]

$$R_{sd}^{\mathrm{OST}} = \sum_{k=1}^{M+1} f_k C_d(\mathcal{A}_d^{(k)}) = \max_{\{M,\mathbf{a},\mathbf{f}\}} \left(\min_{m=1,\ldots,M+1} \sum_{k=1}^{m} f_k C_{a_{m+1}}(\mathcal{A}_d^{(k)}) \right) \tag{8}$$

subject to $\sum_{m=1}^{M+1} f_m = 1$, where we recall that the subset $\mathcal{A}_d^{(m)} \subseteq \mathcal{A}_d$ contains the first m nodes in vector a. In (8), with a slight abuse of notation, we have denoted by the same letters both the variables derived from the OST algorithm defined above (left-hand side) and the variables subject to optimization in the right-hand side. Moreover, the second equality in (8) is easily proved by noticing that the max-min problem at hand prescribes an optimal solutions where nodes are activated "as soon as possible" (i.e., without any further delay after successful decoding) so as not to create bottlenecks along the route. In [6], it is proved through random coding arguments that the rate (8) is achievable under the assumption that channel state information is available only at the receiving end of each wireless link.

Comparing the rate (8) with (4), it easy to demonstrate that, since the collaborative capacity $C_{a_{m+1}}(\mathcal{A}_d^{(m)})$ is larger than $C_{a_{m+1}}(a_m)$ for any m, then (distributed) OST outperforms MH, in the sense that OST provides a larger achievable rate.

4. COLLABORATIVE COMMUNICATIONS IN AD HOC NETWORKS: SPATIAL REUSE

Here we extend the analysis presented in the previous section to the case where multiple, say Q, concurrent source-destination transmissions $\{s_j, d_j\}_{j=1}^{Q}$ are active simultaneously (spatial reuse). In this case, performance comparison between different techniques has to be based on the evaluation of the (Q-dimensional) *capacity region*, that is, on the set of rates $\{R_{s_j d_j}\}_{j=1}^{Q}$ achievable by the given transmission scheme. As discussed below, it is not possible to draw a definite conclusion about the relationship between the capacity regions of (centralized) MH and (distributed) OST, as in the case where only one source-destination transmission is allowed. In particular, the diversity and power gains of OST are here counterbalanced by the increased interference level on concurrent transmissions due to the opportunistic transmission of idle nodes. In the following, this problem is outlined and analyzed by extending the treatment of the previous section.

In Section 5, a framework proposed in [2] for the numerical evaluation of capacity regions is reviewed and extended to OST. This discussion will enable the numerical results presented in Section 6.

4.1. *Optimal multihop transmission*

Following the discussion in the previous section, with spatial reuse, the set of active nodes in each time period $[\sum_{j=1}^{m-1} f_j, \sum_{j=1}^{m-1} f_j + f_m)$ is $\mathcal{A}^{(m)} = \bigcup_{j=1}^{Q} \mathcal{A}_{d_j}^{(m)}$, where each set $\mathcal{A}_{d_j}^{(m)}$ contains the index of the node (if any) relaying the data stream to destination d_j. Clearly, optimality of the schedule cannot be defined univocally as in the scenario without spatial reuse, since here there are Q data rates $R_{s_j d_j}$ as performance measures. The analysis has to rely on the derivation of the capacity region, that is, of the set of achievable rates $\{R_{s_j d_j}\}_{j=1}^{Q}$. Using the same notation as in the previous section, the rates $\{R_{s_j d_j}\}_{j=1}^{Q}$ are achievable if there exist an integer $0 \le M \le n-2$, an $(M+1) \times 1$ vector $\mathbf{f}$, and a sequence of sets $\mathcal{A}_{d_j}^{(m)}$ such that ($j = 1, \ldots, Q$):

$$R_{s_j d_j}^{\mathrm{MH}} \le \min_{m \in \mathcal{M}_{d_j}} f_m C_{\mathcal{A}_{d_j}^{(m+1)}}(\mathcal{A}_{d_j}^{(m)}), \tag{9}$$

where $\mathcal{M}_{d_j}$ is the set of indices $m = 1, \ldots, M$ such that $\mathcal{A}_{d_j}^{(m)}$ is not empty. A computational framework that allows to derive numerically the capacity region of ad hoc networks employing MH has been presented in [5] based on linear programming, and is briefly reviewed in Section 5.1.

4.2. *Opportunistic space-time cooperation*

Similar to the case of no spatial reuse treated in Section 3.2, here all Q sources s_k start transmitting at rates $R_{s_k d_k}$, not being informed of whether the signal will arrive to the destination directly or by collaborative transmission. All idle nodes listen to the transmissions. As soon as a node manages to decode one of the signals from any of the sources, while treating the others as interference, it starts transmitting.

To elaborate, the first node a_2 that is able to decode a signal by any source s_k, treating the others as interference, will start cooperating with s_k. Similar to (5), the time instant of this first decoding can be computed as ($\mathcal{A}_{d_k}^{(1)} = \{s_k\}$ and $\mathcal{A}^{(1)} = \cup_{k=1}^{Q} \mathcal{A}_{d_k}^{(1)}$),

$$f_1 = \min_{k=1,\ldots,Q;\, a_2 \in \mathcal{N} \setminus \mathcal{A}^{(1)}} \frac{R_{s_k d_k}}{C_{a_2}(s_k)}, \tag{10}$$

where the minimum has to be taken with respect to both the pair index k and to the node index a_2. The signal radiated by a_2 cooperates for the decoding of the signal transmitted by s_k but, on the other hand, increases the interference for the reception of the signals of the remaining sources. At this point, define $\mathcal{A}_{d_k}^{(2)} = \{s, a_2\}$ and $\mathcal{A}_{d_j}^{(2)} = \mathcal{A}_{d_k}^{(2)}$ for $j \neq k$. If a third node a_3 is now able to decode the signal from any source s_j, possibly different from s_k, it starts collaborating and the procedure repeats. Similar to (7), the time of activation of the cooperating nodes can be written as

$$f_m = \min_{j=1,\ldots,Q;\, a_{m+1} \in \mathcal{N} \setminus \mathcal{A}^{(m)}} \frac{R_{s_j d_j} - \sum_{k=1}^{m-1} f_k C_{a_{m+1}}(\mathcal{A}_{d_j}^{(k)}, \mathcal{A}^{(k)})}{C_{a_{m+1}}(\mathcal{A}_{d_j}^{(m)}, \mathcal{A}^{(m)})}, \quad m = 1, \ldots, M \tag{11}$$

with $\sum_{m=1}^{M} f_m < 1$. Therefore, the rates achieved by OST are

($f_{M+1} = 1 - \sum_{m=1}^{M} f_m$),

$$R^{\mathrm{OST}}_{s_j d_j} = \sum_{k=1}^{M+1} f_k C_{d_j}\big(\mathcal{A}^{(k)}_{d_j}, \mathcal{A}^{(k)}\big), \quad j = 1, \dots, Q. \tag{12}$$

These rates can be shown to be achievable through random coding following the same arguments as in [6].

As opposed to the case of no spatial reuse (see (8)), the greedy procedure described above cannot be written as the solution of an optimization problem. The reason is that in the former scenario, any new transmission does not generate interference, and, therefore, activating new nodes is only advantageous to the system performance. On the other hand, when spatial reuse is allowed, newly activated nodes not only support the communication of one source-pair destination but also interfere with the other concurrent transmissions. In general, the centralized control of interference carried out by MH may yield a larger capacity region for a transmission protocol that allows spatial reuse. In order to compare the performance of MH and OST with spatial reuse, we have then to resort to the framework presented in [2] for the derivation of capacity regions. More comments on this performance comparison based on numerical results will be presented in Section 6.

5. CAPACITY REGION WITH COLLABORATIVE COMMUNICATIONS

In this section, we first review the framework presented in [2] for the numerical calculation of capacity regions with MH (Section 5.1) and then extend the idea to include OST (Section 5.2). For a related analysis of the case where single-relay AF cooperation is considered, the reader is referred to [4]. The tools developed in this section will be employed in Section 6 to get insight into the performance of the OST scheme.

5.1. *Capacity region and uniform capacity*

The basic concept in the framework of [2] is that of a *basic transmission scheme*, which describes a possible state of the ad hoc network under the considered transmission protocol. For instance, in the case of a protocol that allows MH transmission with no spatial reuse, each transmission scheme is characterized by a transmitter i and a receiver j communicating on behalf of a source node s. The number of available transmission schemes is thus $\breve{M} = n \cdot n(n-1) + 1$, where n is the number of possible source nodes and $n(n-1)$ is the number of transmitting-receiving pairs. More generally, if MH and spatial reuse are allowed, every basic transmitting scheme is characterized by a set of active nodes $\mathcal{A}$ and the corresponding set of receiving nodes $\mathcal{R}$, where mapping between $\mathcal{A}$ and $\mathcal{R}$ is one-to-one. Therefore, the number of basic transmission schemes reads $\breve{M} = \sum_{i=1}^{\lfloor n/2 \rfloor} n^i \cdot (n!/i!(n-2i)!) + 1$ [2].

Each basic transmission scheme, say the mth, is mathematically characterized by a $n \times n$ basic rate matrix $\mathbf{R}_m$, defined as ($s, k = 1, \dots, n$ and $m = 1, \dots, \breve{M}$):

$$R_{m,sk} = \begin{cases} C_k(i, \mathcal{A}), & \text{if node } k \in \mathcal{R} \text{ receives from any} \\ & i \in \mathcal{A}, \text{ with } s \text{ as the source node,} \\ -C_j(k, \mathcal{A}), & \text{if node } k \in \mathcal{A} \text{ transmits to any} \\ & j \in \mathcal{R}, \text{ with } s \text{ as the source node,} \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Let us define an $n \times n$ nonnegative matrix $\mathbf{R}$, with R_{sd} being the rate between a source s and a destination d ($s, d = 1, \dots, n$). The rates in $\mathbf{R}$ are achievable (i.e., $\mathbf{R}$ belongs to the capacity region, see definition in Section 4.1) if there exists an $\breve{M} \times 1$ vector $\mathbf{f} = [f_1 \cdots f_{\breve{M}}]^T$ such that

$$\mathbf{R} = \sum_{m=1}^{\breve{M}} f_m \mathbf{R}_m \quad \text{with} \quad \sum_{m=1}^{\breve{M}} f_m = 1. \tag{14}$$

Similar to the previous sections, the elements in $\mathbf{f}$ define the fraction of time where the corresponding basic transmission scheme is employed in the time-division schedule that realizes the rates in $\mathbf{R}$. Notice that, as stated in the introduction, achieving the points on the boundary of the capacity region requires a (centralized) optimization of the time schedule vector $\breve{\mathbf{f}}$.

In order to employ a single quantity characterizing the performance of a network, [5] defines the *uniform capacity* as the maximum rate simultaneously achievable over all the $n(n-1)$ wireless links of the network. A rate R is uniformly achievable by the network if and only if the rate matrix $\mathbf{R}$ with $R_{ij} = R$ for $i \neq j$ belongs to the capacity region (14). The (per node) uniform capacity R_u is the maximum rate uniformly achievable by the network.

5.2. *Application to opportunistic space-time cooperation*

With OST, a basic transmission scheme is identified by a given choice of source-destination pairs $\{s_j, d_j\}_{j=1}^{Q}$. In fact, for each set of source-destination pairs, the achievable rates are uniquely defined by (8) and (12) for $Q = 1$ (no spatial reuse), and any Q (spatial reuse), respectively.

Starting with the case of no spatial reuse, there are $\breve{M} = n(n-1) + 1$ basic transmission schemes and corresponding basic rate matrices $\{\mathbf{R}_m\}_{m=1}^{\breve{M}}$ of size $n \times n$, corresponding to all the pairs of source-destination nodes. In particular, each transmission scheme is characterized by a source s and a destination d, and the basic rate matrix reads

$$R_{m,ij} = \begin{cases} R^{\mathrm{OST}}_{sd}, & \text{for } i = s, j = d \text{ (see (8)),} \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

Notice that no negative elements are prescribed since multihop is not allowed.

On the other hand, if we consider spatial reuse, each transmission scheme is characterized by $Q = 1, \dots, \lfloor n/2 \rfloor$

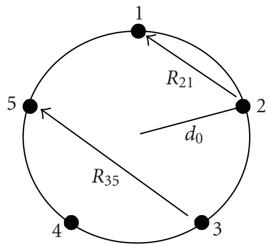

FIGURE 3: The ring network topology. Communication rates R_{21} and R_{35} are shown for reference.

source-destination pairs $\{s_k, d_k\}_{k=1}^{Q}$. Since there are $n!/(Q! \cdot (n-2Q)!)$ distinct choices for the Q source-destination pairs, the number of basic transmission schemes reads $\check{M} = \sum_{k=1}^{\lfloor n/2 \rfloor} n!/[Q!(n-2Q)!]+1$. Moreover, the basic rate matrix for the transmission scheme characterized by source-destination pairs $\{s_k, d_k\}_{k=1}^{Q}$ reads

$$R_{m,ij} = \begin{cases} R_{s_k d_k}^{\mathrm{OST}}, & \text{for } i = s_k,\, j = d_k \text{ for } k = 1, \ldots, Q \text{ (see (12))}, \\ 0, & \text{otherwise}. \end{cases} \tag{16}$$

Notice that, as opposed to MH, the vertices of the capacity region with OST correspond to a given transmission mode and do not require centralized optimization of the time schedule.

6. NUMERICAL RESULTS

In this section, we present numerical results in order to corroborate the analysis presented throughout the paper. The considered scenario is the ring network in Figure 3 with bandwidth $B = 1$ MHz, noise power spectral density $N_0 = -100$ dBm/Hz, reference distance equal the radius of the network $d_0 = 10$ m, path loss exponent $\alpha = 4$, transmitted power $P = 20$ dBm.

Toward the goal of getting insight into the performance comparison between (centralized) optimal MH and (distributed) OST, we first consider an AWGN scenario, that is, with no fading ($|h_{ij}|^2 = 1$ in (1)) in Figure 4. The constant ρ_0 in (1) is set so that the average SNR at d_0 with no interference is 0 dB (i.e., $\rho_0 P/N_0 = 0$ dB). A slice of the capacity region corresponding to rates R_{21} and R_{35} is shown in Figure 4. Let us consider the case of no spatial reuse. As a reference, the capacity regions for (i) single-hop transmission; (ii) single-relay AF collaboration [4] are shown. As proved in Section 3, the capacity region of OST is larger than that of MH due to the power gain that node 3 can capitalize upon by collaborating with both nodes 4 and 2 while communicating with 5 through OST.

Considering now the spatial reuse scenario, from the discussion in Section 4, it is expected that OST should perform at its best for "localized" and low-rate communications. This is because (i) "long-range" (i.e., with source and destination being far apart) communications tend to create a large amount of interference due to the OST mechanism; (ii) high-rate communications set a stringent requirement on the

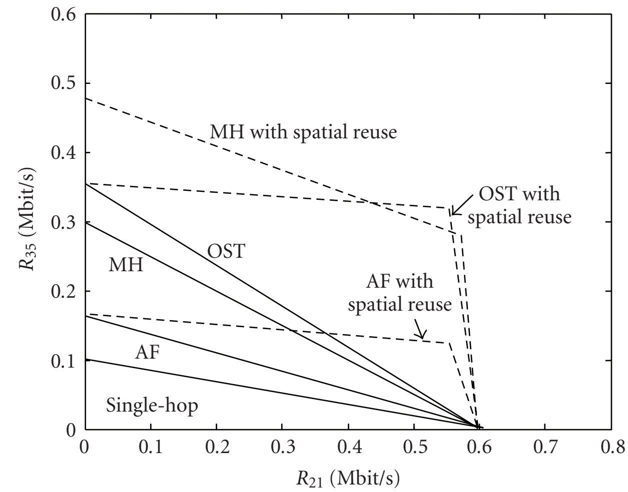

FIGURE 4: Capacity regions slices in the plane R_{21} versus R_{35} for different transmission protocols.

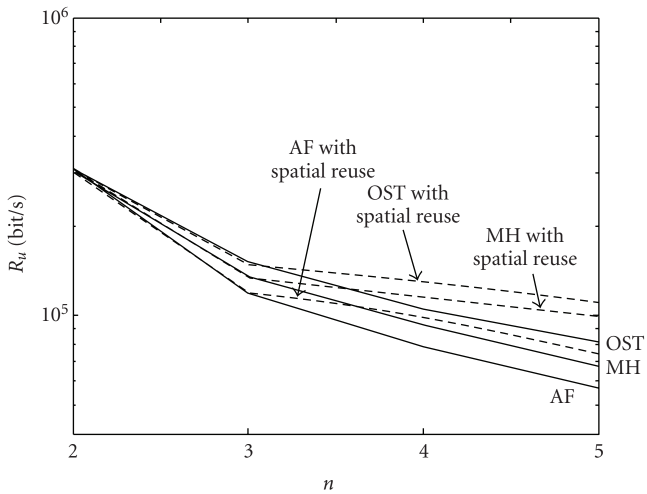

FIGURE 5: Per node uniform capacity R_u versus the number of nodes n for different transmission protocols.

interference level of the network which is difficult to meet through OST (but it is easily controlled through centralized MH). Figure 4 confirms this conclusion in that (i) the capacity region with MH is significantly wider than with OST for large values of R_{35}, where the communication pair 3–5 clearly represents the "nonlocalized" link in the network; (ii) by limiting the rate R_{35} (say $R_{35} < 0.36$), OST can become even more advantageous than MH as a final remark, we notice that, as warned in [4], adding single-relay AF communications to MH does not increase the capacity regions.

In order to evaluate the impact of fading, we consider the (per node) uniform capacity R_u (recall Section 5.1), averaged over the distribution of fading. To account for a fading margin, the average SNR at d_0 with no interference is set here to 10 dB ($\rho_0 P/N_0 = 10$ dB). Figure 5 shows the uniform capacity versus the number of nodes n for a ring network (the case

$n = 5$ is illustrated in Figure 4). Without spatial reuse, as expected, the uniform capacity of OST is superior to MH (for $n = 5$, the gain is approximately 15% for the range of considered n). Moreover, OST is advantageous even in the case of spatial reuse (up to 11% for $n = 5$). This can be explained following the same lines as above since the uniform capacity accounts for a fair condition where all the nodes get to transmit at the same (low) rate towards all possible receivers.

7. CONCLUDING REMARKS

In this paper, the distributed scheme proposed in [6] for opportunistic collaborative communication (OST) has been investigated as an alternative to optimal centralized resource allocation through multihop (MH) in wireless ad hoc networks. The main conclusion is that, while OST always outperforms MH if no spatial reuse is allowed, in a scenario with spatial reuse, applicability of OST is limited to local and low-rate connections due to the distributed interference generated by the opportunistic mechanism of OST. Performance of OST is studied according to the achievable rates obtained in [6] by assuming random coding. Therefore, the results herein have to be interpreted as a theoretical upper bound on the performance that motivates further research on designing practical coding schemes, such as the overlay coding technique based on convolutional coding presented in [8].

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