The Rest-Frame Instant Form of Dynamics and Dirac's Observables.

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Since our understanding of both general relativity and the standard model of elementary particles either with or without supersymmetry is based on singular Lagrangians, whose associated Hamiltonian formalism requires Dirac-Bergmann theory of constraints[1], it is very difficult to identify which are the physical degrees of freedom to be used in the description and interpretation of the fundamental interactions. This is the problem of the physical observables in gauge theories and general relativity.

While behind the gauge freedom of gauge theories proper there are Lie groups acting on some internal space so that the measurable quantities must be gauge invariant, the gauge freedom of theories invariant under diffeomorphism groups of the underlying spacetime (general relativity, string theory and reparametrization invariant systems of relativistic particles) concerns the arbitrariness for the observer in the choice of the definition of "what is space and/or time" (and relative times in the case of particles), i.e. of the definitory properties either of spacetime itself or of the measuring apparatuses. This is the classical mathematical background on which our understanding of the quantum field theory of electromagnetic, weak and strong interactions in the modern BRS formulation is based. The same is true for our attempts to build quantum gravity notwithstanding our actual incapacity to reconcile the influence of gravitational physics on the existence and formulation of spacetime concepts with the basic ideas of quantum theory, which requires a given absolute background spacetime.

In Minkowski spacetime, one uses the covariant approach based on the BRS symmetry which, at least at the level of the algebra of infinitesimal gauge transformations, allows a regularization and renormalization of the relevant theories inside the framework of local quantum field theory. However, problems like the understanding of finite gauge transformations and of the associated moduli spaces, the Gribov ambiguity dependence on the choice of the function space for the fields and the gauge transformations, the confinement of quarks, the definition of relativistic bound states and how to put them among the asymptotic states, the nonlocality of charged states in quantum electrodynamics, not to speak of the conceptual and practical problems posed by gravity, suggest that we should revisit the foundations of our theories.

When we will succeed to reformulate classical physics in terms of the physical degrees of freedom hidden behind manifest gauge and/or general covariance, the quantization of the resulting theories will require to abandon local field theory at the non-perturbative level and to understand how to regularize and renormalize the Coulomb and radiation gauges of electrodynamics to start with.

I revisited the classical Hamiltonian formulation of theories described by singular Lagrangians trying to choose the mathematical frameworks which at each step looked more natural to clarify the physical interpretational problems by means of the use of suitable adapted coordinates. In particular, after many years of dominance of the point of view privileging manifest Lorentz, gauge and general covariance at the price of loosing control on the physical degrees of freedom and on their deterministic evolution (felt as a not necessary luxury only source of difficulties and complications), I went back to the old concept of Dirac observables, namely of those gauge invariant deterministic variables which describe a canonical basis of measurable quantities for the electromagnetic, weak and strong interactions in Minkowski spacetime. Instead, in general relativity, due to the problem of the individuation of the points of spacetime, measurable quantities have a more complex identification, which coincides with Dirac's observables (in any case indispensable for the treatment of the Cauchy problem) only in a completely fixed gauge (total breaking of general covariance).

See Ref.[2] for a complete review of what has been understood till now in the Hamiltonian framework of constraint theory, where the physical observables of gauge theories are identified with the gauge invariant Dirac observables (this definition can be extended to theories invariant under diffeomorphisms like general relativity, but I will not speak about this topic in this short review).

In the search of physical observables there are two primary problems:

A) The choice of the function space in which our classical field theory is defined. The existence of the Gribov ambiguity in gauge theories with the associated 'cone over cone' structure of singularities and stratification of both the space of solutions of the equations of motion and of the associated constraint manifold in phase space depends on such a choice. If the Gribov ambiguity is only a mathematical obstruction one must work in special weighted Sobolev spaces where it is absent. If, instead, there is some physics hidden into it, one works in ordinary Sobolev spaces, but one has to face the above singularities and stratification, which can constitute an obstruction already at the classical level for the construction of global Dirac observables. When this type of singularities can be avoided, the search of Dirac observables is done by solving the multitemporal equations and by looking for special Shanmugadhasan canonical trasformations[3] to find special Darboux bases adapted to the constraints of the theory and containing as a subset a canonical basis of Dirac observables[2, 4].

In the finite dimensional case general theorems connected with the Lie theory of function groups ensure the existence of local Shanmugadhasan canonical transformations from the original canonical variables q^i, p_i, in terms of which the first class constraints (assumed globally defined) have the form $\phi_\alpha(q,p) \approx 0$, to canonical bases P_α, Q_α, P_A, Q_A, such that the equations $P_\alpha \approx 0$ locally define the same original constraint manifold (the P_α are an Abelianization of the first class constraints); the Q_α are the adapted Abelian gauge variables describing the gauge orbits (they are a realization of the times τ_α of the multitemporal equations in terms of variables q^i, p_i); the Q_A, P_A are an adapted canonical basis of Dirac observables. These canonical transformations are the basis of the Hamiltonian definition of the Faddeev-Popov measure of the path integral[5] and give a trivialization of the BRS construction of observables (the BRS method works when the first class constraints may be Abelianized[6]).

Putting equal to zero the Abelianized gauge variables one defines a local gauge of the model. If a system with constraints admits one (or more) global Shanmugadhasan canonical transformations, one obtains one (or more) privileged global gauges in which the physical Dirac observables are globally defined and globally separated from the gauge degrees of freedom [for systems with a compact configuration space Q this is generically impossible]. These privileged gauges (when they exist) can be called generalized Coulomb gauges.

When there is reparametrization invariance of the original action $S = \int dtL$, the canonical Hamiltonian vanishes and both kinematics and dynamics are contained in the first class constraints describing the system: these can be interpreted as generalized Hamilton-Jacobi equations, so that the Dirac observables turn out to be the Jacobi data. When there is a kinematical symmetry group, like the Galileo or Poincaré groups, an evolution may be reintroduced by using the energy generator as Hamiltonian.

Inspired by Ref.[7] the canonical reduction to noncovariant generalized Coulomb gauges, with the determination of the physical Hamiltonian as a function of a canonical basis of Dirac's observables, has been achieved for the following isolated systems (for them one asks that the 10 conserved generators of the Poincaré algebra are finite so to be able to use group theory):

1) Relativistic particle mechanics. Its importance stems from the fact that quantum field theory has no particle interpretation: this is forced on it by means of the asymptotic states of the reduction formalism which correspond to the quantization

of independent one-body systems described by relativistic mechanics. After a study of the one-body problems corresponding to the basic wave equations, the two-body DrozVincent-Todorov-Komar model [see Ref.[2] for the related bibliography] with an arbitrary action-at-a-distance interaction instantaneous in the rest frame has been completely understood both at the classical and quantum level [8]. Its study led to the identification of a class of canonical transformations (utilizing the standard Wigner boost for timelike Poincaré orbits) which allowed to understand how to define suitable center-of-mass and relative variables (in particular a suitable relative energy is determined by a combination of the two first class constraints, so that the relative time variable is a gauge variable), how to find a quasi-Shanmugadhasan canonical transformation adapted to the constraint determining the relative energy, how to separate the four, topologically disjoined, branches of the mass spectrum (it is determined by the other independent combination of the constraints; therefore, there is a distinct Shanmugadhasan canonical transformation for each branch). At the quantum level it was possible to find four physical scalar products, compatible with both the resulting coupled wave equations (i.e. independent from the relative and the absolute rest-frame times): they have been found as generalization of the two existing scalar products of the Klein-Gordon equation: all of them are non-local even in the limiting free case and differ among themselves for the sign of the norm of states on different mass-branches. This example shows that the physical scalar product knows the functional form of the constraints. The wave functions of the quantized model can be obtained from the solutions of a Bethe-Salpeter equation with the same instantaneous potential by multiplication for a delta function containing the relative energy to exclude its spurious solutions (non physical excitations in the relative energy). The extension of the model to two pseudoclassical electrons and to an electron and a scalar has been done and the first was used to get good fits to meson spectra.

The previous canonical transformations were then extended to N free particles described by N mass-shell first class constraints $p_i^2 - m_i^2 \approx 0$: N-1 suitable relative energies are determined by N-1 combinations of the constraints (so that the conjugate N-1 relative times are gauge variables), while the remaining combination determines the 2^N branches of the mass spectrum. The N gauge freedoms associated with these N combinations of the first class constraints are the freedom of the observer: i) in the choice of the time parameter to be used for the overall evolution of the isolated system; ii) in the choice of the description of the relative motions with any given delay among the pairs of particles.

2) Both the open and closed Nambu string[9].

3) Yang-Mills theory with Grassmann-valued fermion fields [10] in the case of a trivial principal bundle over a fixed-x^o R^3 slice of Minkowski spacetime with suitable Hamiltonian-oriented boundary conditions; this excludes monopole solutions (to have them, even if they have been not yet found experimentally, one needs a nontrivial bundle and a variational principle formulated on the bundle, because the gauge potentials on Minkowski spacetime are not globally defined) and, since R^3 is

not compactified, one has only winding number and no instanton number. After a discussion of the Hamiltonian formulation of Yang-Mills theory, of its group of gauge transformations and of the Gribov ambiguity, the theory has been studied in suitable weighted Sobolev spaces where the Gribov ambiguity is absent [11] and the global color charges are well defined. The global Dirac observables are the transverse quantities $\vec{A}_{a\perp}(\vec{x}, x^o)$, $\vec{E}_{a\perp}(\vec{x}, x^o)$ and fermion fields dressed with Yang-Mills (gluonic) clouds. The nonlocal and nonpolynomial (due to the presence of classical Wilson lines along flat geodesics) physical Hamiltonian has been obtained: it is nonlocal but without any kind of singularities, it has the correct Abelian limit if the structure constants are turned off, and it contains the explicit realization of the abstract Mitter-Viallet metric.

4) The Abelian and non-Abelian SU(2) Higgs models with fermion fields[12], where the symplectic decoupling is a refinement of the concept of unitary gauge. There is an ambiguity in the solutions of the Gauss law constraints, which reflects the existence of disjoint sectors of solutions of the Euler-Lagrange equations of Higgs models. The physical Hamiltonian and Lagrangian of the Higgs phase have been found; the self-energy turns out to be local and contains a local four-fermion interaction.

5) The standard SU(3)xSU(2)xU(1) model of elementary particles[13] with Grassmann-valued fermion fields. The final reduced Hamiltonian contains nonlocal self-energies for the electromagnetic and color interactions, but "local ones" for the weak interactions implying the nonperturbative emergence of 4-fermions interactions.

B) All the physical systems defined in the flat Minkowski spacetime, have the global Poincare' symmetry. This suggests to study the structure of the constraint manifold from the point of view of the orbits of the Poincare' group. If p^μ is the total momentum of the system, the constraint manifold has to be divided in four strata (some of them may be absent for certain systems) according to whether $p^2 > 0$, $p^2 = 0$, $p^2 < 0$ or $p^\mu = 0$. Due to the different little groups of the various Poincare' orbits, the gauge orbits of different sectors will not be diffeomorphic. Therefore the constraint manifold is a stratified manifold and the gauge foliations of relativistic systems are nearly never nice, but rather one has to do with singular foliations. For an acceptable relativistic system the stratum $p^2 < 0$ has to be absent to avoid tachyons. To study the strata $p^2 = 0$ and $p^\mu = 0$ one has to add these relations as extra constraints.

For all the strata the next step is to do a canonical transformation from the original variables to a new set consisting of center-of-mass variables x^μ, p^μ and of variables relative to the center of mass. Let us now consider the stratum $p^2 > 0$. By using the standard Wigner boost $L^\mu_\nu(p, \overset{\circ}{p})$ ($p^\mu = L^\mu_\nu(p, \overset{\circ}{p})\overset{\circ}{p}{}^\nu$, $\overset{\circ}{p}{}^\mu = \eta\sqrt{p^2}(1; \vec{0})$,

$\eta = sign\, p^o$), one boosts the relative variables at rest. The new variables are still canonical and the base is completed by p^μ and by a new center-of-mass coordinate $\tilde{x}^\mu$, differing from x^μ for spin terms. The variable $\tilde{x}^\mu$ has complicated covariance properties; instead the new relative variables are either Poincare' scalars or Wigner spin-1 vectors, transforming under the group O(3)(p) of the Wigner rotations induced by the Lorentz transformations. A final canonical transformation[8], leaving fixed the relative variables, sends the center-of-mass coordinates $\tilde{x}^\mu$, p^μ in the new set $p \cdot \tilde{x}/\eta\sqrt{p^2} = p \cdot x/\eta\sqrt{p^2}$ (the time in the rest frame), $\eta\sqrt{p^2}$ (the total mass), $\vec{k} = \vec{p}/\eta\sqrt{p^2}$ (the spatial components of the 4-velocity $k^\mu = p^\mu/\eta\sqrt{p^2}$, $k^2 = 1$), $\vec{z} = \eta\sqrt{p^2}(\vec{\tilde{x}} - \tilde{x}^o\vec{p}/p^o)$. $\vec{z}$ is a noncovariant center-of-mass canonical 3-coordinate multiplied by the total mass: it is the classical analog of the Newton-Wigner position operator (like it, $\vec{z}$ is covariant only under the little group O(3)(p) of the timelike Poincaré orbits). Analoguous considerations could be done for the other sectors.

The nature of the relative variables depends on the system. The first class constraints, once rewritten in terms of the new variables, can be manipulated to find suitable global and Lorentz scalar Abelianizations. Usually there is a combination of the constraints which determines $\eta\sqrt{p^2}$, i.e. the mass spectrum, so that the time in the rest frame $p \cdot x/\eta\sqrt{p^2}$ is the conjugated Lorentz scalar gauge variable. The other constraints eliminate some of the relative variables (in particular the relative energies for systems of interacting relativistic particles and the string): their conjugated coordinates (the relative times) are the other gauge variables: they are identified with a possible set of time parameters by the multitemporal equations. The Dirac observables (apart from the center-of-mass ones $\vec{k}$ and $\vec{z}$) have to be extracted from the remaining relative variables and the construction shows that they will be either Poincare' scalars or Wigner covariant objects. In this way in each stratum preferred global Shanmugadhasan canonical transformations are identified, when no other kind of obstruction to globality is present inside the various strata.

To covariantize the description of the previous reduced systems, which is valid in Minkowski spacetime with Cartesian coordinates, again the starting point was given by Dirac[1] with his reformulation of classical field theory on spacelike hypersurfaces foliating Minkowski spacetime M^4 [the foliation is defined by an embedding $R \times \Sigma \rightarrow M^4$, $(\tau, \vec{\sigma}) \mapsto z^{(\mu)}(\tau, \vec{\sigma}) \in \Sigma_\tau$, with Σ an abstract 3-surface diffeomorphic to R^3, with Σ_τ its copy embedded in M^4 labelled by the value τ (the Minkowski flat indices are (μ); the scalar "time" parameter τ labels the leaves of the foliation, $\vec{\sigma}$ are curvilinear coordinates on Σ_τ and $(\tau, \vec{\sigma})$ are Σ_τ-adapted holonomic coordinates for M^4); this is the classical basis of Tomonaga-Schwinger quantum field theory]. In this way one gets a parametrized field theory with a covariant 3+1 splitting of Minkowski spacetime and already in a form suited to the transition to general relativity in its ADM canonical formulation. The price is that one has to add as new independent configuration variables the embedding coordinates $z^{(\mu)}(\tau, \vec{\sigma})$ of the points of the spacelike hypersurface Σ_τ [the only ones carrying Lorentz indices] and then to define the fields on Σ_τ so that they know the hypersurface Σ_τ of τ-simultaneity [for a Klein-Gordon field $\phi(x)$, this new field is $\tilde{\phi}(\tau, \vec{\sigma}) = \phi(z(\tau, \vec{\sigma}))$: it contains the nonlocal information about the embedding]. Then one rewrites the Lagrangian of the given isolated system in the form required by the coupling to an external gravitational field, makes the previous 3+1 splitting of Minkowski spacetime and interpretes all the fields of the system as the new fields on Σ_τ (they are Lorentz scalars, having only surface indices). Instead of considering the 4-metric as describing a gravitational field (and therefore as an independent field as it is done in metric gravity, where one adds the Hilbert action to the action for the matter fields), here one replaces the 4-metric with the the induced metric $g_{AB}[z] = z^{(\mu)}_A \eta_{(\mu)(\nu)} z^{(\nu)}_B$ on Σ_τ [a functional of $z^{(\mu)}$; here we use the notation $\sigma^A = (\tau, \sigma^r)$; $z^{(\mu)}_A = \partial z^{(\mu)}/\partial\sigma^A$ are flat tetrad fields on Minkowski spacetime with the $z^{(\mu)}_r$'s tangent to Σ_τ] and considers the embedding coordinates $z^{(\mu)}(\tau, \vec{\sigma})$ as independent fields [this is not possible in metric gravity, because in curved spacetimes $z^\mu_A \neq \partial z^\mu/\partial\sigma^A$ are not tetrad fields so that holonomic coordinates $z^\mu(\tau, \vec{\sigma})$ do not exist]. From this Lagrangian, besides a Lorentz-scalar form of the constraints of the given system, we get four extra primary first class constraints

$$\mathcal{H}_{(\mu)}(\tau, \vec{\sigma}) = \rho_{(\mu)}(\tau, \vec{\sigma}) - l_{(\mu)}(\tau, \vec{\sigma}) T^{\tau\tau}_{sys}(\tau, \vec{\sigma}) - z_{r(\mu)}(\tau, \vec{\sigma}) T^{\tau r}_{sys}(\tau, \vec{\sigma}) \approx 0$$

[here $T^{\tau\tau}_{sys}(\tau, \vec{\sigma})$, $T^{\tau r}_{sys}(\tau, \vec{\sigma})$, are the components of the energy-momentum tensor in the holonomic coordinate system, corresponding to the energy- and momentum-density of the isolated system; one has $\{\mathcal{H}_{(\mu)}(\tau, \vec{\sigma}), \mathcal{H}_{(\nu)}(\tau, \vec{\sigma}^{'})\} = 0$] implying the independence of the description from the choice of the 3+1 splitting, i.e. from the choice of the foliation with spacelike hypersufaces.

In special relativity, it is convenient to restrict ourselves to arbitrary spacelike hyperplanes $z^{(\mu)}(\tau, \vec{\sigma}) = x^{(\mu)}_s(\tau) + b^{(\mu)}_r(\tau)\sigma^r$. Since they are described by only 10 variables, after this restriction we remain only with 10 first class constraints determining the 10 variables conjugate to the hyperplane in terms of the variables of the system:

$$\mathcal{H}^{(\mu)}(\tau) = p^{(\mu)}_s - p^{(\mu)}_{(sys)} \approx 0,\ \mathcal{H}^{(\mu)(\nu)}(\tau) = S^{(\mu)(\nu)}_s - S^{(\mu)(\nu)}_{(sys)} \approx 0.$$

The 20 variables for the phase space description of a hyperplane are:
i) $x^{(\mu)}_s(\tau), p^{(\mu)}_s$, parametrizing the origin of the coordinates on the family of spacelike hyperplanes. The four constraints $\mathcal{H}^{(\mu)}(\tau) \approx 0$ say that $p^{(\mu)}_s$ is determined by the 4-momentum of the isolated system.
ii) $b^{(\mu)}_A(\tau)$ (with the $b^{(\mu)}_r(\tau)$'s being three orthogonal spacelike unit vectors generating the fixed τ-independent timelike unit normal $b^{(\mu)}_\tau = l^{(\mu)}$ to the hyperplanes) and $S^{(\mu)(\nu)}_s = -S^{(\nu)(\mu)}_s$ with the orthonormality constraints $b^{(\mu)}_A\, {}^4\eta_{(\mu)(\nu)} b^{(\nu)}_B = {}^4\eta_{AB}$ [enforced by assuming the Dirac brackets $\{S^{(\mu)(\nu)}_s, b^{(\rho)}_A\} = {}^4\eta^{(\rho)(\nu)} b^{(\mu)}_A - {}^4\eta^{(\rho)(\mu)} b^{(\nu)}_A$, $\{S^{(\mu)(\nu)}_s, S^{(\alpha)(\beta)}_s\} = C^{(\mu)(\nu)(\alpha)(\beta)}_{(\gamma)(\delta)} S^{(\gamma)(\delta)}_s$ with $C^{(\mu)(\nu)(\alpha)(\beta)}_{(\gamma)(\delta)}$ the structure constants of the Lorentz algebra]. In these variables there are hidden six independent pairs of degrees of freedom. The six constraints $\mathcal{H}^{(\mu)(\nu)}(\tau) \approx 0$ say that $S^{(\mu)(\nu)}_s$ coincides the spin tensor of the isolated system. Then one has that $p^{(\mu)}_s$, $J^{(\mu)(\nu)}_s = x^{(\mu)}_s p^{(\nu)}_s - x^{(\nu)}_s p^{(\mu)}_s + S^{(\mu)(\nu)}_s$,

satisfy the algebra of the Poincaré group.

Let us remark that, for each configuration of an isolated system there is a privileged family of hyperplanes (the Wigner hyperplanes orthogonal to $p_s^{(\mu)}$, existing when $p_s^2 > 0$) corresponding to the intrinsic rest-frame of the isolated system. If we choose these hyperplanes with suitable gauge fixings, we remain with only the four constraints $\mathcal{H}^{(\mu)}(\tau) \approx 0$, which can be rewritten as

$$\sqrt{p_s^2} \approx [invariant\, mass\, of\, the\, isolated\, system\, under\, investigation] = M_{sys};$$
$$\vec{p}_{sys} = [3 - momentum\, of\, the\, isolated\, system\, inside\, the\, Wigner\, hyperplane] \approx 0.$$

There is no more a restriction on $p_s^{(\mu)}$, because $u_s^{(\mu)}(p_s) = p_s^{(\mu)}/p_s^2$ gives the orientation of the Wigner hyperplanes containing the isolated system with respect to an arbitrary given external observer.

In this special gauge we have $b_A^{(\mu)} \equiv L^{(\mu)}{}_A(p_s, \overset{\circ}{p}_s)$ (the standard Wigner boost for timelike Poincaré orbits), $S_s^{(\mu)(\nu)} \equiv S_{system}^{(\mu)(\nu)}$, and the only remaining canonical variables are the noncovariant Newton-Wigner-like canonical "external" center-of-mass coordinate $\tilde{x}_s^{(\mu)}(\tau)$ (living on the Wigner hyperplanes) and $p_s^{(\mu)}$. Now 3 degrees of freedom of the isolated system [an "internal" center-of-mass 3-variable $\vec{\sigma}_{sys}$ defined inside the Wigner hyperplane and conjugate to $\vec{p}_{sys}$] become gauge variables [the natural gauge fixing is $\vec{\sigma}_{sys} \approx 0$, so that it coincides with the origin $x_s^{(\mu)}(\tau) = z^{(\mu)}(\tau, \vec{\sigma} = 0)$ of the Wigner hyperplane], while the $\tilde{x}^{(\mu)}$ is playing the role of a kinematical external center of mass for the isolated system and may be interpreted as a decoupled observer with his parametrized clock (point particle clock). All the fields living on the Wigner hyperplane are now either Lorentz scalar or with their 3-indices transformaing under Wigner rotations (induced by Lorentz transformations in Minkowski spacetime) as any Wigner spin 1 index.

One obtains in this way a new kind of instant form of the dynamics (see Ref.[15]), the "Wigner-covariant 1-time rest-frame instant form"[16] with a universal breaking of Lorentz covariance. It is the special relativistic generalization of the nonrelativistic separation of the center of mass from the relative motion $[H = \frac{\vec{P}^2}{2M} + H_{rel}]$. The role of the center of mass is taken by the Wigner hyperplane, identified by the point $\tilde{x}^{(\mu)}(\tau)$ and by its normal $p_s^{(\mu)}$. The invariant mass M_{sys} of the system replaces the nonrelativistic Hamiltonian H_{rel} for the relative degrees of freedom, after the addition of the gauge-fixing $T_s - \tau \approx 0$ [identifying the time parameter τ, labelling the leaves of the foliation, with the Lorentz scalar time of the center of mass in the rest frame, $T_s = p_s \cdot \tilde{x}_s / M_{sys}$; M_{sys} generates the evolution in this time].

The determination of $\vec{\sigma}_{sys}$ may be done with the group theoretical methods of Ref.[17]: given the "internal" realization on the phase space of a given system of the ten Poincaré generators [it is determined by the previous constraints] one can build three 3-position variables only in terms of them, which in our case of a system on the Wigner hyperplane with $\vec{p}_{sys} \approx 0$ are: i) a canonical center of mass (the "internal" center of mass $\vec{\sigma}_{sys}$); ii) a noncanonical Møller center of energy $\vec{\sigma}_{sys}^{(E)}$; iii) a noncanonical Fokker-Pryce center of inertia $\vec{\sigma}_{sys}^{(FP)}$. Due to $\vec{p}_{sys} \approx 0$, we have $\vec{\sigma}_{sys} \approx \vec{\sigma}^{(E)}_{sys} \approx \vec{\sigma}^{(FP)}_{sys}$. By adding the gauge fixings $\vec{\sigma}_{sys} \approx 0$ one can show that the origin $x^{(\mu)}_s(\tau)$ becomes simultaneously the Dixon center of mass of an extended object and both the Pirani and Tulczyjew centroids (see Ref. [18] for the application of these methods to find the center of mass of a configuration of the Klein-Gordon field after the preliminary work of Ref.[19]). With similar methods one can construct three "external" collective positions (all located on the Wigner hyperplane) from the rest-frame instant form realization of the "external" Poincaré group: i) the "external" canonical noncovariant center of mass $\tilde{x}^{(\mu)}_s$; ii) the "external" noncanonical and noncovariant Møller center of energy $R^{(\mu)}_s$; iii) the "external" covariant noncanonical Fokker-Pryce center of inertia $Y^{(\mu)}_s$ (when there are the gauge fixings $\vec{\sigma}_{sys} \approx 0$ it also coincides with the origin $x^{(\mu)}_s$). It turns out that the Wigner hyperplane is the natural setting for the study of the Dixon multipoles of extended relativistic systems[20] and for defining the canonical relative variables with respect to the center of mass. The Wigner hyperplane with its natural Euclidean metric structure offers a natural solution to the problem of boost for lattice gauge theories and realizes explicitly the machian aspect of dynamics that only relative motions are relevant.

The isolated systems till now analyzed to get their rest-frame Wigner-covariant generalized Coulomb gauges [i.e. the subset of global Shanmugadhasan canonical bases, which, for each Poincaré stratum, are also adapted to the geometry of the corresponding Poincaré orbits with their little groups; these special bases can be named Poincaré-Shanmugadhasan bases for the given Poincaré stratum of the presymplectic constraint manifold (every stratum requires an independent canonical reduction); till now only the main stratum with p^2 timelike and $W^2 \neq 0$ has been investigated] are:

a) The system of N scalar particles with Grassmann electric charges plus the electromagnetic field [16]. The starting configuration variables are a 3-vector $\vec{\eta}_i(\tau)$ for each particle [$x^{(\mu)}_i(\tau) = z^{(\mu)}(\tau, \vec{\eta}_i(\tau))$] and the electromagnetic gauge potentials $A_A(\tau, \vec{\sigma}) = \frac{\partial z^{(\mu)}(\tau,\vec{\sigma})}{\partial \sigma^A} A_{(\mu)}(z(\tau, \vec{\sigma}))$, which know the embedding of Σ_τ into M^4. One has to choose the sign of the energy of each particle, because there are not mass-shell constraints (like $p_i^2 - m_i^2 \approx 0$) among the constraints of this formulation, due to the fact that one has only three degrees of freedom for particle, determining the intersection of a timelike trajectory and of the spacelike hypersurface Σ_τ. For each choice of the sign of the energy of the N particles, one describes only one of the 2^N branches of the mass spectrum of the manifestly covariant approach based on the coordinates $x^{(\mu)}_i(\tau)$, $p^{(\mu)}_i(\tau)$, i=1,..,N, and on the constraints $p_i^2 - m_i^2 \approx 0$ (in the free case). In this way, one gets a description of relativistic particles with a given sign of the energy with consistent couplings to fields and valid independently from the quantum effect of pair production [in the manifestly covariant approach, containing all possible branches of the particle mass spectrum, the classical counterpart of pair production is the intersection of different branches deformed by the presence of interactions]. The final Dirac's observables are: i) the transverse radiation field variables $\vec{A}_\perp$, $\vec{E}_\perp$; ii) the particle canonical variables $\vec{\eta}_i(\tau)$, $\vec{\kappa}_i(\tau)$, dressed with a Coulomb cloud. The physical Hamiltonian contains the mutual instantaneous Coulomb potentials extracted from field theory and there is a regularization of the Coulomb self-energies due to the Grassmann character of the electric charges Q_i $[Q_i^2 = 0]$. In Ref.[21] there is the study of the Lienard-Wiechert potentials and of Abraham-Lorentz-Dirac equations in this rest-frame Coulomb gauge and also scalar electrodynamics is reformulated in it. Also the rest-frame 1-time relativistic statistical mechanics has been developed [16]. The extraction of the Darwin potential from the Lienard-Wiechert solution is nearly accomplished [22]. A general study of the relativistic center od mass, of the rotational kinematics and of Dixon multipolar expansions[20] is under investigation[23].

b) The system of N scalar particles with Grassmann-valued color charges plus the color SU(3) Yang-Mills field[24]: it gives the pseudoclassical description of the relativistic scalar-quark model, deduced from the classical QCD Lagrangian and with the color field present. The physical invariant mass of the system is given in terms of the Dirac observables. From the reduced Hamilton equations the second order equations of motion both for the reduced transverse color field and the particles are extracted. Then, one studies the N=2 (meson) case. A special form of the requirement of having only color singlets, suited for a field-independent quark model, produces a "pseudoclassical asymptotic freedom" and a regularization of the quark self-energy. With these results one can covariantize the bosonic part of the standard model given in Ref.[13].

c) The system of N spinning particles of definite energy $[(\frac{1}{2},0)$ or $(0,\frac{1}{2})$ representation of SL(2,C)] with Grassmann electric charges plus the electromagnetic field[25] and that of a Grassmann-valued Dirac field plus the electromagnetic field (the pseudoclassical basis of QED) [26]. In both cases there are geometrical complications connected with the spacetime description of the path of electric currents and not only of their spin structure, suggesting a reinterpretation of the supersymmetric scalar multiplet as a spin fibration with the Dirac field in the fiber and the Klein-Gordon field in the base; a new canonical decomposition of the Klein-Gordon field into center-of-mass and relative variables [19, 18] will be helpful to clarify these problems. After their solution and after having obtained the description of Grassmann-valued chiral fields the rest-frame form of the full standard $SU(3) \times SU(2) \times U(1)$ model can be achieved.

d) The study of the definition of collective and relative variables for the Klein-Gordon field, initiated in Refs.[19], has been reformulated in the rest-frame instant form[18] with a discussion of how to find the canonical internal center of mass of the field configuration. Also Dixon's multipolar expansions are studied at the Hamiltonian level on the Wigner hyperplanes. Now the same problematic is under investigation for the electromagnetic field[27].

e) The relativistic perfect fluids[28].

As shown in Refs.[16, 10], the rest-frame instant form of dynamics automatically gives a physical ultraviolet cutoff in the spirit of Dirac and Yukawa: it is the Møller radius[29] $\rho = \sqrt{-W^2}/p^2 = |\vec{S}|/\sqrt{p^2}$ ($W^2 = -p^2\vec{S}^2$ is the Pauli-Lubanski Casimir when $p^2 > 0$), namely the classical intrinsic radius of the worldtube, around the covariant noncanonical Fokker-Pryce center of inertia $Y^{(\mu)}$, inside which the non-covariance of the canonical center of mass $\tilde{x}^\mu$ is concentrated. At the quantum level ρ becomes the Compton wavelength of the isolated system multiplied its spin eigenvalue $\sqrt{s(s+1)}$, $\rho \mapsto \hat{\rho} = \sqrt{s(s+1)}\hbar/M = \sqrt{s(s+1)}\lambda_M$ with $M = \sqrt{p^2}$ the invariant mass and $\lambda_M = \hbar/M$ its Compton wavelength. Therefore, the criticism to classical relativistic physics, based on quantum pair production, concerns the testing of distances where, due to the Lorentz signature of spacetime, one has intrinsic classical covariance problems: it is impossible to localize the canonical center of mass $\tilde{x}^\mu$ adapted to the first class constraints of the system (also named Pryce center of mass and having the same covariance of the Newton-Wigner position operator) in a frame independent way. For more details see Ref.[2].

In conclusion, the best set of canonical coordinates adapted to the constraints and to the geometry of Poincaré orbits in Minkowski spacetime and naturally predisposed to the coupling to canonical tetrad gravity is emerging for the electromagnetic, weak and strong interactions with matter described either by fermion fields or by relativistic particles with a definite sign of the energy.

After having studied the canonical reduction in the 3-orthogonal gauge of a new formulation of tetrad gravity[2, 30] and its deparametrization to the rest-frame instant form of dynamics in Minkowski spacetime when the Newton constant is switched off, the main tasks for the future are:

A) Make the canonical quantization of scalar electrodynamics in the rest-frame instant form on the Wigner hyperplanes, which should lead to a particular realization of Tomonaga-Schwinger quantum field theory, avoiding the no-go theorems of Refs.[31]. The main problems to be investigated are
i) the use of the wave functions of the quantization of positive energy scalar particles[32] to define Tomonaga-Schwinger asymptotic states and a LSZ reduction formalism [since Fock states do not constitute a Cauchy problem for the field equations, because an in (or out) particle can be in the absolute future of another one due to the tensor product nature of these asymptotic states, bound state equations like the Bethe-Salpeter one have spurious solutions which are excitations in relative energies, the variables conjugate to relative times]. Moreover, it should be possible to include bound states among the asymptotic states.
ii) The search of a Schroedinger-like equation for bound states by using the Schwinger-Dyson equations but avoiding the Bethe-Salpeter equation with its spurious solutions [2] [see Refs.[33, 34] for the nonrelativistic case).
iii) Use the Møller radius as a physical ultraviolet cutoff for the point splitting technique.
iv) See how to use the results of Refs.[35] about the infrared dressing of asymptotic states in S matrix theory to avoid the 'infraparticle' problem[36].

B) Study the linearization of tetrad gravity in the 3-orthogonal gauge to reformulate the theory of gravitational waves in this gauge.

C) Study the N-body problem in tetrad gravity at the lowest order in the Newton constant (action-at-a-distance plus linearized tetrad gravity).

D) Begin the study of the standard model of elementary particles coupled to tetrad gravity starting from the Einstein-Maxwell system.

References

[1] P.A.M.Dirac, Can.J.Math. **2**, 129 (1950); "Lectures on Quantum Mechanics", Belfer Graduate School of Science, Monographs Series (Yeshiva University, New York, N.Y., 1964). J.L.Anderson and P.G.Bergmann, Phys.Rev. **83**, 1018 (1951). P.G.Bergmann and J.Goldberg, Phys.Rev. **98**, 531 (1955).

[2] L.Lusanna, "Towards a Unified Description of the Four Interactions in Terms of Dirac-Bergmann Observables", invited contribution to the book of the Indian National Science Academy for the International Mathematics Year 2000 AD (hep-th/9907081).

[3] S.Shanmugadhasan, J.Math.Phys. **14**, 677 (1973).

[4] L.Lusanna, Phys.Rep. **185**, 1 (1990). Riv. Nuovo Cimento **14**, n.3, 1 (1991). J.Math.Phys. **31**, 2126 (1990). J.Math.Phys. **31**, 428 (1990). Int.J.Mod.Phys. **A8**, 4193 (1993). M.Chaichian, D.Louis Martinez and L.Lusanna, Ann.Phys.(N.Y.)**232**, 40 (1994).

[5] L.D.Faddeev and Popov, Phys.Lett. **B25**, 30 (1967).

[6] M.Henneaux, Phys.Rep. **126**, 1 (1985).

[7] P.A.M.Dirac, Can.J.Phys. **33**, 650 (1955).

[8] G.Longhi and L.Lusanna, Phys.Rev. **D34**, 3707 (1986).

[9] F.Colomo, G.Longhi and L.Lusanna, Int.J.Mod.Phys. **A5**, 3347 (1990); Mod.Phys.Letters **A5**, 17 (1990). F.Colomo and L.Lusanna, Int.J.Mod.Phys. **A7**, 1705 and 4107 (1992).

[10] L.Lusanna, Int.J.Mod.Phys. **A10**, 3531 and 3675 (1995).

[11] V.Moncrief, J.Math.Phys. **20**, 579 (1979). M.Cantor, Bull.Am.Math.Soc. **5**, 235 (1981).

[12] L.Lusanna and P.Valtancoli, Int.J.Mod.Phys. **A12**, 4769 (1997) (hep-th/9606078) and Int.J.Mod.Phys. **A12**, 4797 (1997). (hep-th/9606079).

[13] L.Lusanna and P.Valtancoli, Int.J.Mod.Phys. **A13**, 4605 (1998) (hep-th/9707072).

[14] K.Kuchar, J.Math.Phys. **17**, 777, 792, 801 (1976); **18**, 1589 (1977).

[15] P.A.M.Dirac, *Rev.Mod.Phys.* **21** (1949) 392.

[16] L.Lusanna, Int.J.Mod.Phys. **A12**, 645 (1997).

[17] M.Pauri and M.Prosperi, J.Math.Phys. **16**, 1503 (1975).

[18] L.Lusanna and M.Materassi, "The Canonical Decomposition in Collective and Relative Variables of a Klein-Gordon Field in the Rest-Frame Wigner-Covariant Instant Form", Firenze Univ.preprint (hep-th/9904202).

[19] G.Longhi and M.Materassi, J.Math.Phys. **40**, 480 (1999) (hep-th/9803128); "Collective and Relative Variables for a Classical Klein-Gordon Field", Firenze Univ.preprint (hep-th/9890024), to appear in Int.J.Mod.Phys. A.

[20] W.G.Dixon, J.Math.Phys. **8**, 1591 (1967). "Extended Objects in General Relativity: their Description and Motion", in "Isolated Gravitating Systems in General Relativity", ed.J.Ehlers (North-Holland, Amsterdam, 1979).

[21] D.Alba and L.Lusanna, Int.J.Mod.Phys. **A13**, 2791 (1998) (hep-th/9705155).

[22] H.Crater and L.Lusanna, "The Rest-Frame Darwin Potential from the Lienard-Wiechert Solution in the Radiation Gauge", in preparation.

[23] D.Alba, L.Lusanna and M.Pauri, in preparation.

[24] D.Alba and L.Lusanna, Int.J.Mod.Phys. **A13**, 3275 (1998) (hep-th/9705156).

[25] F.Bigazzi and L.Lusanna, Int.J.Mod.Phys. **A14**, 1429 (1999) (hep-th/9807052).

[26] F.Bigazzi and L.Lusanna, Int.J.Mod.Phys. **A14**, 1877 (1999) (hep-th/9807054).

[27] L.Assenza and L.Lusanna, in preparation.

[28] L.Lusanna and D.Nowak-Szczepaniak, in preparation.

[29] C.Møller, Ann.Inst.H.Poincaré **11**, 251 (1949); "The Theory of Relativity" (Oxford Univ.Press, Oxford, 1957).

[30] L.Lusanna and S.Russo, "Tetrad Gravity I): A New Formulation", Firenze Univ. preprint 1998 (gr-qc/9807073). "Tetrad Gravity II): Dirac's Observables", Firenze Univ. preprint 1998 (gr-qc/9807074). R.DePietri and L.Lusanna, "Tetrad Gravity III): Asymptotic Poincaré Charges, the Physical Hamiltonian and Void Spacetimes", (gr-qc/9909025). R.DePietri, L.Lusanna and M.Vallisneri, "Tetrad Gravity IV): The N-body Problem", in preparation.

[31] C.G.Torre and M.Varadarajan, Phys.Rev. **D58**, 064007 (1998); Class.Quantum Grav. **16**, 2651 (1999).

[32] C.Lämmerzahl, J.Math.Phys. **34**, 3918 (1993).

[33] S.S.Schweber, Ann.Phys.(N.Y.) **20**, 61 (1962).

[34] L.Lusanna, "From Relativistic Mechanics towards Green's Functions: Multitemporal Dynamics", Proc. of the VII Seminar on Problems of High Energy Physics and Quantum Field Theory, Protvino 1984.

[35] R.Horan, M.Lavelle and D.McMullan, Pramana **51**, 317 (1998); "Asymptotic Dynamics in Quantum Field Theory", hep-th/9909044. M.Lavelle and D.McMullan, Phys.Rep. **279**, 1 (1997).

[36] D.Buchholz, Phys.Lett. **B174**, 331 (1986).