

# SUPERNOVA NEUTRINOS: STRONG COUPLING EFFECTS OF WEAK INTERACTIONS

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## ABSTRACT

In core-collapse supernovae,  $\nu$  and  $\bar{\nu}$  are initially subject to significant self-interactions induced by weak neutral currents, which may induce strong-coupling effects on the flavor evolution (collective transitions). The interpretation of the effects is simplified when self-induced collective transitions are decoupled from ordinary matter oscillations, as for the matter density profile that we discuss. In this case, approximate analytical tools can be used (pendulum analogy, swap of energy spectra). For inverted  $\nu$  mass hierarchy, the sequence of effects involves: synchronization, bipolar oscillations, and spectral split. Our simulations shows that the main features of these regimes are not altered when passing from simplified (angle-averaged) treatments to full, multi-angle numerical experiments.

## 1. Prologue

Densely packed individuals often show a surprising, collective behavior (Fig. 1). Recent developments suggest that neutrinos make no exception, despite the weakness of their self-interactions. Effects of  $\nu$ - $\nu$  forward scattering (via neutral currents) may be as important as the known effects of  $\nu_e$ - $e^-$  forward scattering in matter (via charged currents), provided that the  $\nu$  number density is very high. The dense core of exploding Supernovae might provide a possible environment where the flavor evolution of  $\nu$ 's and  $\bar{\nu}$ 's can show, indeed, highly nonlinear and strongly coupled effects.



Figure 1: In a school of fish, individuals often show a collective behavior. Very dense neutrino gases, like those emerging from a core-collapse supernova, might show analogous features in flavor space.

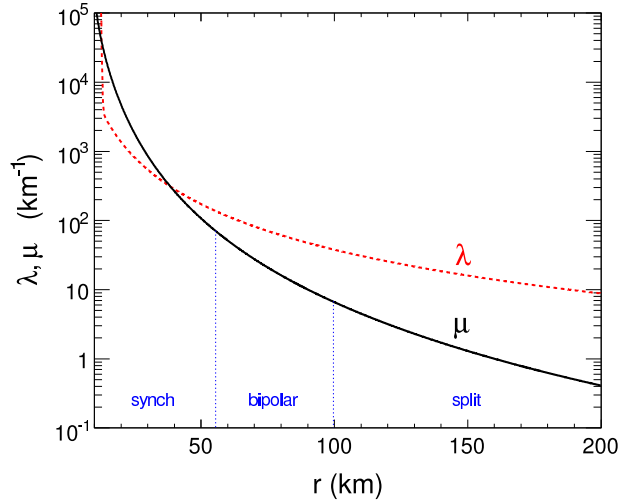


Figure 2: Radial profiles of the neutrino self-interaction parameter  $\mu(r) = \sqrt{2} G_F (N + \bar{N})$  and of the matter-interaction parameter  $\lambda(r) = \sqrt{2} G_F N_{e^-}$  adopted in this work, in the range  $r \in [10, 200]$  km.

## 2. Reference supernova model

Supernova  $\nu$  oscillations are a very important tool to study astrophysical processes and to better understand  $\nu$  properties<sup>1)</sup>. After leaving the neutrinosphere,  $\nu$  and  $\bar{\nu}$  undergo flavor oscillations triggered by vacuum mass-mixing parameters and by ordinary (MSW) matter effects. Besides, in the first few hundred kilometers neutrino-neutrino interactions may induce additional important effects (depending on the neutrino mass hierarchy). Self-interaction effects are expected to be non negligible when  $\mu(r) \sim \omega$ , where  $\mu = \sqrt{2} G_F (N_\nu(r) + \bar{N}_\nu(r))$  is the potential associated to the  $\nu + \bar{\nu}$  background [analogous to the MSW potential  $\lambda = \sqrt{2} G_F N_{e^-}(r)$ ], while  $\omega = \Delta m^2 / E$  is the largest vacuum oscillation frequency. We neglect the smallest mass squares difference  $\delta m^2 = m_2^2 - m_1^2 \ll \Delta m^2 = |m_3^2 - m_{1,2}^2|$ , and consider a  $2\nu$  mixing scenario governed by  $\Delta m^2$  and the mixing angle  $\theta_{13}$  ( $\Delta m^2 = 10^{-3}$  eV<sup>2</sup> and  $\sin^2 \theta_{13} = 10^{-4}$  for reference). In the supernova context,  $\nu_\mu$  and  $\nu_\tau$  (shortly,  $\nu_x$ ) behave similarly, and we can generically consider two-neutrino  $\nu_e \leftrightarrow \nu_x$  oscillations as a reasonable approximation.

Figure 1 shows the radial profiles of the matter potential  $\lambda(r)$  and of the neutrino potential  $\mu(r)$ , and the approximate ranges where different collective effects occur: synchronization, bipolar oscillation and spectral split. The nonlinearity of the self interactions induce collective transitions for small  $r$ , well before the ordinary MSW resonance, allowing a clear interpretation of the numerical simulations. However, for matter profiles different from ours (shallow electron density profiles<sup>2)</sup>), the MSW effects can be already operative around  $O(100)$  km, in which case (not typical, and not considered here) they are entangled to the collective ones in a complicated way.

We adopt normalized thermal spectra with  $\langle E_e \rangle = 10$  MeV,  $\langle \bar{E}_e \rangle = 15$  MeV, and  $\langle E_x \rangle = \langle \bar{E}_x \rangle = 24$  MeV for  $\nu_e$ ,  $\bar{\nu}_e$ ,  $\nu_x$  and  $\bar{\nu}_x$ , respectively. The emission geometry is based on the so called “bulb model”<sup>2)</sup> with spherical symmetry: neutrinos are assumed to be half-isotropically emitted from the neutrinosphere. Along any radial trajectory there is, therefore, a cylindrical symmetry. As cylindrical variables one can choose the distance from the supernova center  $r$ , and the angle  $\vartheta$  between two interacting neutrino trajectories. If the dependence on  $\vartheta$  is integrated out, one speaks of “single-angle” approximation, while the general situation of variable  $\vartheta$  is dubbed “multi-angle” case. The numerical simulation in the multi-angle case is extremely challenging, since it typically requires the solution of a large system ( $\sim 10^M$ ,  $M \geq 5$ ) of coupled non-linear equations, after discretization of the cylindrical coordinates.

### 3. Equations of motion, pendulum analogy, and spectral split

The propagation of neutrinos of given energy  $E$  is studied through the Liouville equation for the  $2 \times 2$  neutrino density matrix in flavor basis. By expanding it on the Pauli and the identity matrices, the equations of motion can be expressed in terms of two flavor polarization vectors,  $\mathbf{P}(E)$  for any neutrino and  $\bar{\mathbf{P}}(E)$  for any antineutrino. By introducing a vector  $\mathbf{B}$  that depends on the mixing angle  $\theta_{13}$ , and a vector  $\mathbf{D} = \mathbf{J} - \bar{\mathbf{J}}$  that is the difference between the integral over the energy of  $\mathbf{P}$  and  $\bar{\mathbf{P}}$ , the equations of motion can be written as,

$$\dot{\mathbf{P}} = (+\omega\mathbf{B} + \lambda\mathbf{z} + \mu\mathbf{D}) \times \mathbf{P} , \quad (1)$$

$$\dot{\bar{\mathbf{P}}} = (-\omega\mathbf{B} + \lambda\mathbf{z} + \mu\mathbf{D}) \times \bar{\mathbf{P}} , \quad (2)$$

see<sup>3)</sup> and references therein. In the general case, the polarization vectors depend also on the neutrino emission angle  $\theta_0$  at the neutrinosphere (the neutrino intersection angle  $\vartheta$  can be expressed in terms of  $r$  and of  $\theta_0$ ). The  $\nu_e$  survival probability  $P_{ee}$  is a function of the flavor polarization  $z$ -component,  $P_{ee} = 1/2(1 + P_f^z/P_z^i)$ , where the  $i$  and  $f$  refer to the initial and final state, respectively (analogously for  $\bar{\nu}_e$ ).

Collective effects show up by aligning such polarization vectors (in flavor space) close to each other. In the alignment approximation, the equations of motion for  $\mathbf{P}(E)$  and  $\bar{\mathbf{P}}(E)$  can be reduced to collective ones describing a classical, gyroscopic pendulum: a spherical pendulum of unit length in a constant gravity field, characterized by a point-like massive bob spinning around the pendulum axis with constant angular momentum<sup>4,5)</sup>. The pendulum inertia is inversely proportional to  $\mu(r)$ , while its angular momentum depends on the difference of the integrated polarization vectors  $\mathbf{J}$  and  $\bar{\mathbf{J}}$ , see<sup>3)</sup>. The motion of a spherical pendulum is, in general, a combination of a precession and a nutation.

In the case of normal neutrino mass hierarchy, the pendulum starts close to the stable, downward position and stays close to it, as  $\mu$  slowly decreases collective effect gradually vanish. In the inverted hierarchy case, the pendulum starts close to the

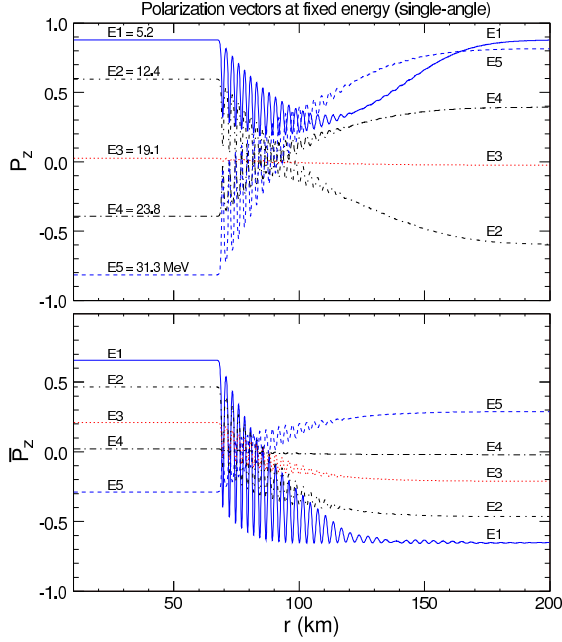


Figure 3: Single-angle simulation in inverted hierarchy:  $P_z$  (neutrinos) and  $\bar{P}_z$  (antineutrinos) as a function of radius, for five energy values.

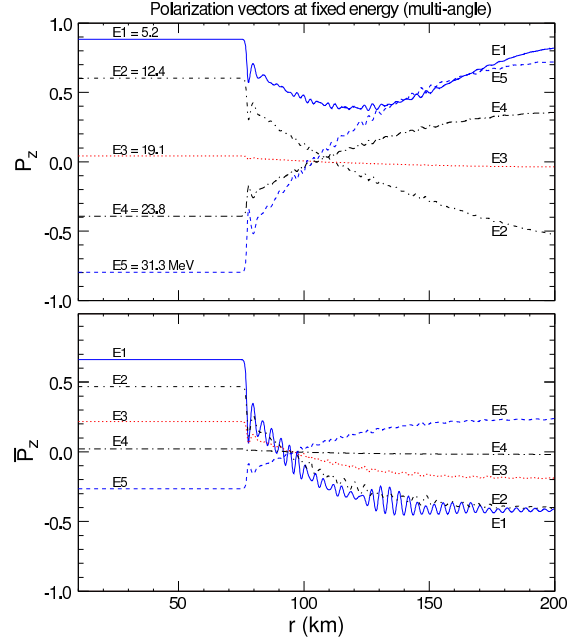


Figure 4: Multi-angle simulation in inverted hierarchy:  $P_z$  (neutrinos) and  $\bar{P}_z$  (antineutrinos) as a function of radius, for five energy values.

“unstable,” upward position, being slightly tilted by an angle of  $O(\theta_{13})$ . At small  $r$ , when  $\mu$  is large (small pendulum inertia), the bob spin dominates and the pendulum remains precessing in the upward position as a “sleeping top”<sup>5)</sup>, a situation named synchronization<sup>6,4)</sup>. As  $\mu$  decreases with  $r$ , the pendulum inertia increases and, unavoidably, for any  $\theta_{13} \neq 0$ , the pendulum fall occurs with subsequent nutations, the so called bipolar oscillations. The increase of the pendulum inertia with  $r$  reduces the amplitude of the nutations, and bipolar oscillations are expected to vanish when self-interaction and vacuum effects are of the same size.

While the bipolar regime comes to an end, self-interaction effects do not completely vanish, and a spectral split builds up: a “stepwise swap” between the  $\nu_e$  and  $\nu_x$  energy spectra. The neutrino swapping can be explained by the conservation of the pendulum “energy” and of the lepton number<sup>7)</sup>. The lepton number conservation is related to the constancy of  $D_z = J_z - \bar{J}_z$ , that is a direct consequence of the equation of motion. For a more detailed description of the pendulum analogy in our reference model, the reader is referred to our work<sup>3)</sup> and references therein.

#### 4. Single- and multi-angle simulations: stability of results

Figures 3 and 4 show the  $z$ -component of  $\mathbf{P}$  and  $\bar{\mathbf{P}}$ , as a function of  $r$  at different  $E$  values, for single- and multi-angle simulations, respectively. Bipolar oscillations start after a synchronization plateau, with equal periods for both  $\nu$  and  $\bar{\nu}$  at any

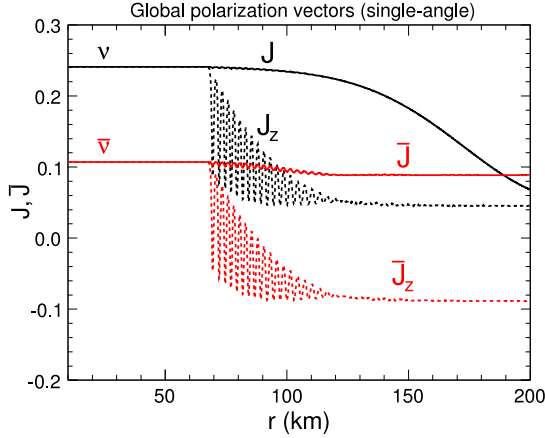


Figure 5: Single-angle simulation in inverted hierarchy: modulus and  $z$ -component of  $\mathbf{J}$  and  $\bar{\mathbf{J}}$ .

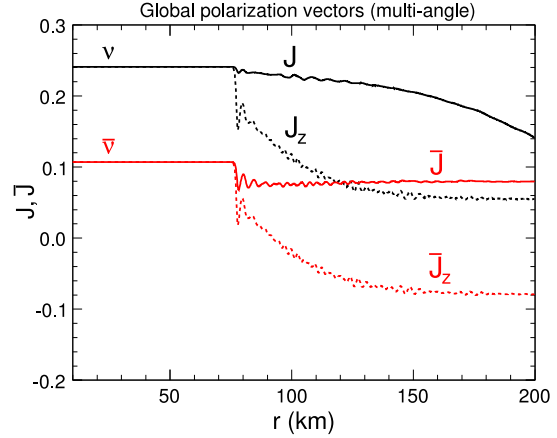


Figure 6: Multi-angle simulation in inverted hierarchy: modulus and  $z$ -component of  $\mathbf{J}$  and  $\bar{\mathbf{J}}$ .

energy, confirming the appearance of collective features. Indeed, the behavior of each  $P_z$  and  $\bar{P}_z$  depends essentially on its energy. For neutrinos, Figure 3, the spectral split (inversion of  $P_z$ ) starts at a critical energy  $E_c \simeq 7$  MeV: the curve relative to  $E < E_c$  ends up at the same initial value ( $P_{ee} = 1$ ), while the curves for  $E > E_c$  show the  $P_z$  inversion ( $P_{ee} = 0$ ). Neutrinos with an energy of  $\sim 19$  MeV do not oscillate much, because this is roughly the energy for which the initial  $\nu_e$  and  $\nu_x$  fluxes are equal in our scenario. For  $\bar{\nu}$ , all curves show almost complete polarization reversal, except at very small energies (of few MeV, not shown). Multi-angle simulations (Fig. 4) are similar, although with somewhat damped bipolar oscillations.

Figures 5 and 6 show the evolution of the global polarization vectors (modulus  $J$  and  $z$ -component  $J_z$ ) for neutrinos and antineutrinos, in the single- and multi-angle cases. The behavior of these vectors can be related to the gyroscopic pendulum motion. At the beginning, in the synchronized regime, all the polarization vectors are aligned so that  $J = J_z$  and  $\bar{J} = \bar{J}_z$ : the pendulum just spins in the upward position without falling. Around  $\sim 70$  km the pendulum falls for the first time and nutations appear. The nutation amplitude gradually decreases and bipolar oscillations eventually vanish for  $r \sim 100$  km. At the same time, the spectral split builds up: antineutrinos tend to completely reverse their polarization, while this happens only partially for neutrinos. As said before, also for antineutrinos there is a partial swap of the spectra for  $E \sim 4$  MeV. From Figure 6 it appears that bipolar oscillations of  $\mathbf{J}$  and  $\bar{\mathbf{J}}$  are largely smeared out in the multi-angle case. The bipolar regime starts somewhat later with respect to the single-angle case, since neutrino-neutrino interaction angles can be larger than the (single-angle) average one, leading to stronger self-interaction effects, that force the system in synchronized mode slightly longer. However, just as in the single-angle case, the spectral split builds up,  $\bar{J}_z$  gets finally reversed, while the difference  $D_z = J_z - \bar{J}_z$  remains constant.

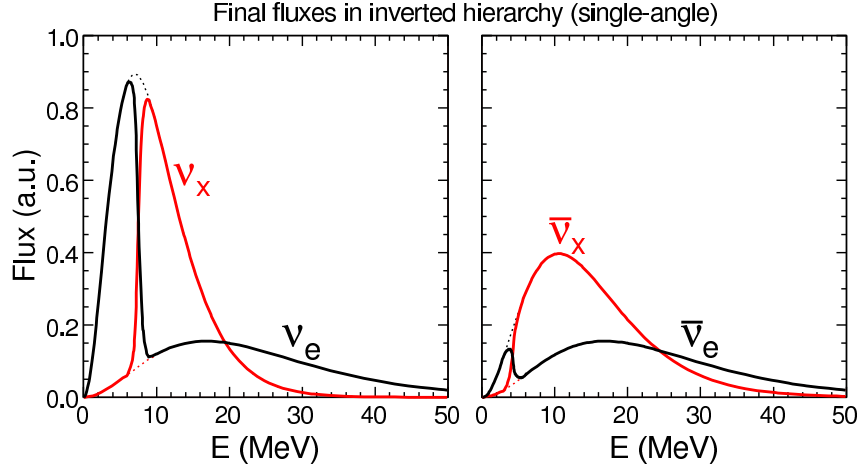


Figure 7: Single-angle simulation in inverted hierarchy: final fluxes (at  $r = 200$  km, in arbitrary units) for different neutrino species as a function of energy. Initial fluxes are shown as dotted lines.

Figures 7 and 8 show the final  $\nu$  and  $\bar{\nu}$  fluxes, in the single- and multi-angle simulations. The  $\nu$  clearly show the spectral split effect and the corresponding sudden swap of  $\nu_e$  and  $\nu_x$  fluxes above  $E_c \simeq 7$  MeV. In the right panel of Figure 7, the final  $\bar{\nu}$  spectra are basically completely swapped with respect to the initial ones, except at very low energies, where there appears a minor  $\bar{\nu}$  spectral split. This phenomenon can be related to the loss of  $\bar{J}$  and of  $|\bar{J}_z|$ <sup>3)</sup>. Also in the multi-angle case of Figure 8, the  $\nu$  spectral swap at  $E > E_c \simeq 7$  MeV is rather evident, although less sharp with respect to the single-angle case, while the minor feature associated to the “antineutrino spectral split” is largely smeared out. We conclude that at least the  $\nu$  spectral split (left panel of either Fig. 7 or 8) provides a robust and potentially observable collective feature emerging in numerical experiments for inverted hierarchy.

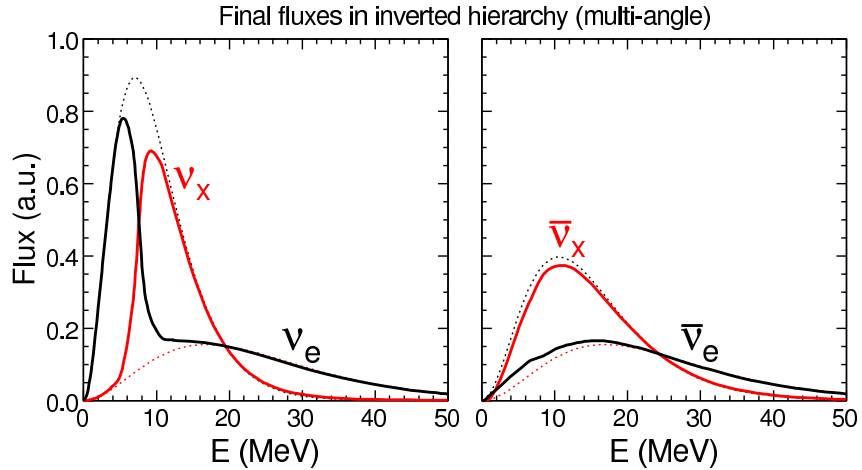


Figure 8: As in Fig. 7, but for multi-angle simulations.

## 5. Summary

We have studied supernova neutrino oscillations in a model where the collective flavor transitions (synchronization, bipolar oscillations, and spectral split) are well separated from later, ordinary MSW effects. We have performed numerical simulations in both single- and multi-angle cases, using continuous energy spectra with significant  $\nu$ - $\bar{\nu}$  and  $\nu_e$ - $\nu_x$  asymmetry. The results of the single-angle simulation can be largely understood by means of an analogy with a classical gyroscopic pendulum. The main observable effect appears to be the swap of final-state energy spectra, for inverted hierarchy, at a critical energy dictated by lepton number conservation. In the multi-angle simulation, details of self-interaction effects can change, but the spectral split remains a robust, observable feature. In this sense, averaging over neutrino trajectories does not alter the main effect of the self interactions. From the point of view of neutrino parameters, collective flavor oscillations in supernovae could be instrumental in identifying the inverse neutrino mass hierarchy, even for tiny  $\theta_{13}$ .<sup>8)</sup>

## 6. Epilogue

Supernova  $\nu$  and  $\bar{\nu}$  flavor polarization vectors can perform elaborate and collective “dances” (precession, nutations) in flavor space, at least for the first  $O(100)$  km, in the case of inverted mass hierarchy. Many aspects of this behavior, however, seem to be precluded to experimental observations, with the possible exception of a robust, finale-state feature: the  $\nu$  spectral split. According to a calculable energy threshold, supernova  $\nu$  might then proceed to the Earth with their original flavor (say,  $\nu_e$ ), or with the complementary one (say,  $\nu_x$ ), just as an initially coherent school of fish (Fig. 1) may finally branch out in some circumstances (Fig. 9).



Figure 9: A school of fish branching out in two different directions. Analogously, supernova neutrino polarization vectors might split up in flavor space, due to self-interaction effects.

## 7. Acknowledgements

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## 8. References

- 1) G. G. Raffelt, “*Supernova neutrino observations: What can we learn?*,” astro-ph/0701677. A very good introduction to the field.
- 2) H. Duan, G. M. Fuller, J. Carlson and Y. Z. Qian, “*Simulation of coherent non-linear neutrino flavor transformation in the supernova environment. I: Correlated neutrino trajectories,*” Phys. Rev. D **74**, 105014 (2006) [astro-ph/0606616]. This seminal paper sparked the recent interest for supernova neutrino self-interactions.
- 3) G. L. Fogli, E. Lisi, A. Marrone and A. Mirizzi, “*Collective neutrino flavor transitions in supernovae and the role of trajectory averaging,*” JCAP **0712**, 010 (2007) [arXiv:0707.1998 [hep-ph]]. See this paper for details not covered in this talk and for a more complete bibliography.
- 4) S. Hannestad, G. G. Raffelt, G. Sigl and Y. Y. Y. Wong, “*Self-induced conversion in dense neutrino gases: Pendulum in flavour space,*” Phys. Rev. D **74**, 105010 (2006) [astro-ph/0608695]. The surprising analogy with a classical pendulum is worked out in this paper.
- 5) H. Duan, G. M. Fuller, J. Carlson and Y. Z. Qian, “*Analysis of Collective Neutrino Flavor Transformation in Supernovae,*” Phys. Rev. D **75**, 125005 (2007) [astro-ph/0703776]. A discussion of several collective effects.
- 6) S. Pastor, G. G. Raffelt and D. V. Semikoz, “*Physics of synchronized neutrino oscillations caused by self-interactions,*” Phys. Rev. D **65**, 053011 (2002) [hep-ph/0109035]. An early discussion of the synchronized (“sleeping top”) regime.
- 7) G. G. Raffelt and A. Y. Smirnov, “*Self-induced spectral splits in supernova neutrino fluxes,*” Phys. Rev. D **76**, 081301 (2007) [Erratum-ibid. D **77**, 029903 (2008)] [arXiv:0705.1830 [hep-ph]]. Analytical demonstration of the spectral split in adiabatic approximation.
- 8) B. Dasgupta, A. Dighe and A. Mirizzi, “*Identifying neutrino mass hierarchy at extremely small  $\theta_{13}$  through Earth matter effects in a supernova signal,*” arXiv:0802.1481 [hep-ph]. Recent discussion of combined collective+Earth effects to probe the mass hierarchy.