Design and Modeling of $\chi^{(2)}$ Second Harmonic Amplification in Circular Photonic Crystal

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ABSTRACT

We analyze in this work the second harmonic amplification of $\chi^{(2)}$ nonlinear process in membrane type GaAs circular photonic crystal. This unconventional kind of photonic crystal is well suited for the generation of whispering gallery modes due to the circular symmetric periodic pattern. The Gaussian beam of a fundamental pump signal at 1.55 μm defines a whispering gallery mode resonance and generates a second harmonic mode at 0.775 μm in the central missing hole micro-cavity. The periodic pattern and the micro-cavity are tailored and optimized in order to generate a second harmonic conversion efficiency of 50 %. We predict the resonances by an accurate 2D time domain model including $\chi^{(2)}$ nonlinearity and also by a 3D Finite Element Method FEM. Moreover, by using a 3D membrane configuration, we predict a quality factor of the second harmonic mode of the order of 35000.

Keywords: Nonlinear Waveguides, Second Harmonic Generation Process, Scalar Hertzian Potential, Circular Photonic Crystal.

INTRODUCTION

Photonic crystals (PhCs) are actually used in many optical applications as biosensors [1], optical resonators [2] and wavelength filters [3]. Several architectures of micro-cavities have been studied in the past by using triangular and square lattices [4],[5], and high performances in micro-cavities have been achieved by both reducing the size of the holes adjacent to the cavity and by shifting their position. In particular the nonlinear properties of PhCs are becoming a new and exciting area of research in nonlinear optics [6]. We analyze in this work the second harmonic amplification of $\chi^{(2)}$ nonlinear process in membrane type circular photonic crystal based on GaAs. This unconventional kind of photonic crystal, reported in Fig. 1, is well suited for the generation of whispering gallery modes due to the circular symmetric periodic pattern. This method can be used for active photonic devices such as micro-cavity lasers with whispering gallery mode (WGM) [7] or circular resonators [8] which usually exhibit excellent characteristics as low losses, good confinement, and high quality Q factor. In our analysis we focus on the study and optimization of the micro-cavity resonance in order to couple and to enhance a second harmonic (SH) signal in the cavity region. In particular, the Gaussian beam of a fundamental pump signal at 1.55 μm defines a whispering gallery mode resonance and generates a second harmonic mode at 0.775 μm in the central missing hole micro-cavity. The periodic pattern and the micro-cavity are tailored and optimized in order to generate a second harmonic conversion efficiency of 50 %.

We predict the resonances by an accurate 2D time domain model including $\chi^{(2)}$ nonlinearity and also by a 3D Finite Element Method (FEM). Indeed, the particular periodic pattern cannot be analyzed with the traditional numerical modelling, such as the wave expansion method in the frequency domain, used often in the modelling of PhC structures: the Brillouin zone is not defined as in triangular or square lattice, and alternative methods such as Finite Difference Time Domain (FDTD) or FEM are necessary.

Focusing on second harmonic generation in $\chi^{(2)}$ nonlinear processes we analyze in this work a GaAs Circular Photonic Crystal Slab (CPhCs) consisting of air holes disposed along periodic concentric lines and along particular patterns. We optimize the radius value and position of all inner holes disposed around the central defect (missing hole) in order to couple a cavity mode corresponding to the second harmonic (SH) signal ($\lambda=0.775 \mu\text{m}$) generated by a laser beam

($\lambda=1.55 \mu\text{m}$) focused on CPhCs. The PhC pattern consists of air holes in a GaAs material along circular concentric lines. These holes have a reduced radius and are shifted starting from the micro-cavity centre and along the radial direction, in order to improve the resonance properties. The shift of the inner holes and the different radius as compared to the outer holes improve the field confinement in the central region by enhancing the cavity modes. A SH coupling efficiency of 50% is checked. Finally we amplify the SH mode, generated by the nonlinear effect, through an air cavity depth below the CPhC by obtaining a Q-factor as high as 35000.

1.1 FEM Modeling and Design

In this section we first characterize the laser beam modeling. When a laser beam propagates it travels as an approximate plane wave with a cross-section intensity of Gaussian shape. The waist of the beam is defined as the Gaussian function's characteristic length. A focused laser beam has a minimum waist (see w_0 in Fig. 2) at a certain point along the direction of propagation. The solution of the time-dependent Maxwell's equations gives the following electric field related to a transverse electric (TE) polarized field (y-components):

$$\mathbf{E}_y(x, y, z, t) = E_0 \frac{w_0}{w(z)} e^{-(r/w(z))^2} \cos\left(\omega t - kz + \eta(z) - r^2 \frac{k}{2R(z)}\right) \mathbf{e}_y \quad (1)$$

where

$$\begin{aligned} w(z) &= w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} \\ \eta(z) &= \text{atan}\left(\frac{z}{z_0}\right) \\ R(z) &= z \left(1 + \left(\frac{z_0}{z}\right)^2\right) \end{aligned} \quad (2)$$

In these expressions, w_0 is the minimum waist, ω is the angular frequency, r is the radial cylindrical coordinate, k is the wave number, and z_0 is the electric field peak position. The wave front of the beam is not exactly planar, it propagates like a spherical wave with radius $R(z)$. Close to the point of the minimum waist the wave is almost plane. In this manner eqs. (1),(2) are used to describe a plane wave at one end of a cylinder (see Fig. 2). In our 2D time domain finite-element method (TD-FEM) [9] the simulated laser signal excites the CPhCs from a side and the equation which represents the wave propagation in the GaAs nonlinear slab is the following:

$$\mu_0 \sigma \frac{\partial \mathbf{A}}{\partial t} + \mu_0 \frac{\partial(\epsilon_0 \epsilon_r \partial \mathbf{A} / \partial t - \mathbf{D})}{\partial t} + \nabla \times (\mu_r^{-1} (\nabla \times \mathbf{A})) = 0 \quad (3)$$

where $\mathbf{A}$ represents the potential vector and $\mathbf{D}$ is given by the constitutive relationship $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$. The nonlinear properties for the SH generation in the nonlinear CPhCs is defined with the following matrix

$$\mathbf{P} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \cdot \begin{bmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_z E_y \\ 2E_z E_x \\ 2E_x E_y \end{bmatrix} \quad (4)$$

where $\mathbf{P}$ is the polarization. The presented time domain model uses, for simplicity, only the d_{11} parameter, according to the TE pump source assumption (in the GaAs case $d_{11}=200\text{pm/V}$ [10]). The thickness T (reported in Fig. 1) of the CPhCs is optimized by introducing the effective dielectric constant (EDC) approach which provides the effective refractive indices of the transverse electric TE slab modes. The Q-factor is evaluated by 3D frequency domain FEM tool [11] which provides the total Q-factor evaluated by considering an air cavity below the PhC slab (membrane-like structure). In particular Q factor is evaluated as [11]

$$Q = 2\pi \text{freq} \frac{U}{P} \quad (5)$$

where U is the energy stored inside the PhCS, P is the power lost through the absorbing material, and freq is the working frequency. The whole structure is embedded in an absorbing material, so it is possible to calculate the ratio between the energy stored (given by the PhCS resonance) and the power loss (flowing outside the structure through the absorbing layer). As FEM boundary condition we consider radiation boundary surfaces in which the second-order radiation boundary condition is used [11]

$$(\nabla \times \mathbf{E})_{\text{tan}} = jk_0 \mathbf{E}_{\text{tan}} - \frac{j}{k_0} \nabla_{\text{tan}} \times (\nabla_{\text{tan}} \times \mathbf{E}_{\text{tan}}) + \frac{j}{k_0} \nabla_{\text{tan}} (\nabla_{\text{tan}} \cdot \mathbf{E}_{\text{tan}}) \quad (6)$$

where $\mathbf{E}_{\text{tan}}$ is the tangential component to the surface of the electric field and j is $(-1)^{1/2}$. We observe that the second-order radiation boundary condition approximates free space, in this way we do not consider the numerical error generated by the backscattering of the boundary condition.

The laser beam is focused in a spot size of $w_0=10\mu\text{m}$, moreover the pump wavelength is $\lambda_0=1.55\mu\text{m}$, the pulse time delay is 40 fs, and the pulse width is 100 fs. The slab thickness and the inner holes of the proposed CPhC are optimized [11] in order to amplify the SH signal generated in the coupled cavity. In particular the optimization is performed by shifting the inner hole position, by changing the inner holes radius and the slab thickness T . In order to analyze the real behaviour of the GaAs slab the real and the imaginary part of the refractive index are considered. The high SH confinement minimizes the absorption effect of the GaAs material. The optimized CPhC is obtained for a lattice constant $a=350\text{nm}$, a inner hole radius $r=100\text{nm}$, a outer hole radius is $R=119\text{nm}$, the distances from the microcavity centre (see Fig. 3) are $d_1=350\text{nm}$, $d_2=360\text{nm}$ and the slab thickness T is 200 nm.

In Fig. 4 the calculated spectra and the electric field profile of the fundamental signal and of the second harmonic signal are reported: the transmittivity is evaluated by the Fast Fourier Transform (FFT) of the total signal (generated by the laser source) inside the cavity region. From Fig. 4 we observe that a good SH ($\lambda_{\text{SH}}=0.775\mu\text{m}$) resonance and coupling is found in the optimized micro-cavity for $T=200\text{ nm}$. For this optimized CPhC, the periodic pattern and the micro-cavity generate a second harmonic conversion efficiency of 50 %. The modal profile of the fundamental and SH modes are reported in Fig. 5. The SH signal, emitted in the vertical y -direction above the microcavity region, can be further amplified by considering an air cavity below the PhC slab such as in the membrane type CPhC reported in Fig. 6. We calculate for the structure of Fig. 6 a SH mode Q factor of 35000 by considering a cavity depth of $h=(3/2)\lambda_{\text{SH}}=1.1625\mu\text{m}$.

2 Conclusion

In this work we predict the presence and the enhancement of a SH field generated by a focused laser beam at $\lambda_0=1.55\mu\text{m}$ in a CPhC micro-cavity. The SH mode ($\lambda=0.775\mu\text{m}$) amplified in the micro-cavity can be used for applications such as add drop filtering and wavelength division multiplexing. In this work we use different numerical tools: the FEM time domain approach which provides the transient analysis and coupling efficiency of the SH signal generated by

the nonlinear process, and the FEM frequency domain approach which provides the total Q factor of the whole membrane PhC type.

Figures and captions.

Fig. 1: Second harmonic generation in nonlinear GaAs PhC slab with circular patterns.

Fig. 2: Focused laser beam profile $w(z)$.

Fig. 3: Inner hole shift regarding the design optimization.

Fig. 4: In plane transmittivity of CPhC for an optimised $T=200$ nm thickness.

Fig. 5: (a) Fundamental WGM mode profile, and (b) SH mode profile.

Fig. 6: (a), (b), (c) Membrane type circular photonic crystal for different angles of view.

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