Applications in Engineering Science 3 (2020) 100013

Contents lists available at ScienceDirect

Applications in Engineering Science

journal homepage: www.elsevier.com/locate/apples

A simplified mechanical model for explaining fast-rising jökulhlaups

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ARTICLE INFO

Keywords:
Subglacial floods
Subglacial lakes
Jökulhlaups
Mathematical modelling

ABSTRACT

The paper describes a simplified mathematical model aimed at fitting available hydrographs of floods from subglacial lakes reaching their peak almost linearly in time, the so-called *fast-rising* jökulhlaups. The simplifying idea is that the glacier can be treated as a block with fixed width, length and thickness, confining a subglacial lake with a constant cross-sectional area and variable level. We do not consider the role of heat transfer as suggested by many authors for fast-rising events. The model consists of two ordinary differential equations: conservation equation for the lake water and motion equation for the glacier. The glacier vertical movement is supposed to be governed by its own weight, the water pressure generated by the lake and by the forces acting on the lateral sides due to interaction between the glacier and the surroundings. The model has three free parameters and reproduces satisfactorily eight historical hydrographs observed originated by eight jökulhlaups in the Skaftá river (Iceland). These floods, of fast-rising type, are sourced from two ice cauldrons, the Eastern one being responsible of the largest floods (up to 3500 $m^3 s^{-1}$). On average, the cauldrons drain almost every two years due to persistent geothermal activity beneath Vatnajökull glacier. This periodicity has a simple mathematical interpretation and is used to obtain another validation of the model.

1. Introduction

Jökulhlaup is an Icelandic word which, in Glaciology, defines a subglacial flood typically from ice-dammed lakes. Such floods are not precisely predictable since too many factors influence the full process. Jökulhlaups have been observed in several glacial areas like Iceland, Greenland, Himalayas, Alps, Antarctica, Alaska, and many other places. Although they often occur in remote areas of the World, these events pose serious hazards for the environment and human beings. This phenomenon is common in Iceland due to the interaction of volcanic activity and glaciers. Indeed, some of the most relevant papers on this subject (e.g. Björnsson, 1974; Nye, 1976), are devoted to floods from Grimsvötn, a subglacial lake under the Vatnajökul glacier in Iceland.

Usually, the flood flows underneath the glacier, travelling over distances of tens of kilometers. Since this area, below hundreds of meters of ice, is not accessible for measurements, the geometry of the drainage system can only be hypothesized and different models have been proposed (a single channel, a network of channels or a porous sheet, see e.g. Nye, 1976; Röthlisberger, 1972; Walder, 1986 and, recently, Hewitt et al., 2012; Schoof et al., 2012). Despite many efforts, a fully satisfactory mathematical model is still challenging and several questions remain only partially answered. Among the main unresolved questions, one has crucial importance: how do jökulhlaups begin? In other words: which are the trigging mechanisms? Answering this would constitute a significant step towards predicting flood size and timing.

Many jökulhlaups occur annually (see Huss et al., 2007) or even a number of times each year (see Roberts et al. (2005)). Sometimes jökulhlaups tend to form a time series with a recurrence period of several years.

Once started, a jökulhlaup reaches the peak discharge in a few days, even a few hours, and ends after one or two weeks; however small lakes may drain within days (see, for example Clarke, 1982). Some authors, as Roberts (2005), categorize the observed hydrographs as *fast-rising* (also termed *linearly-rising*) and *slow-rising* (also termed *exponentially-rising*) depending on dQ/dt, $Q(t)$ being the instantaneous discharge in $m^3 s^{-1}$. Although simple, this choice may be misleading: a better one would be the concave/convex form of the rising hydrograph based on the sign of d^2Q/dt^2. Regardless of a common agreement on how to distinguish slow from fast floods, it is reasonable that an event which reaches its peak in a few hours to a day may be termed as "fast" and can be very dangerous (particularly when $Q_{\max}$ is rather large), allowing a very short time for warning time by local authorities.

In this paper we shall focus only on fast-rising floods and make use of some the available hydrographs from the Eastern Skaftá cauldron.

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https://doi.org/10.1016/j.apples.2020.100013
Received 21 June 2020; Accepted 2 September 2020
Available online 28 September 2020

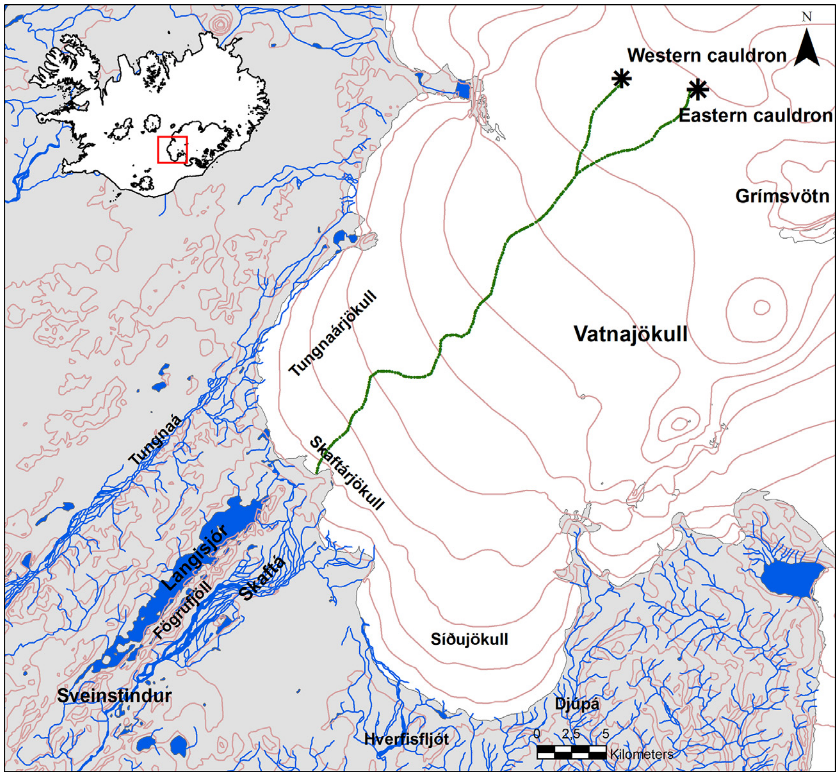

Fig. 1. The eastern (Eystri) and western (Vestari) cauldrons in Vatnajökull glacier. Copyright of this image belongs to Icelandic Meteorological Office.

In the last years other relevant and perhaps more reliable observations have been available to researchers. Among these we mention the effects of the jökulhlaups on the basal sliding of the glacier (see Anderson et al., 2005; Bartholomaus et al., 2007; Einarsson et al., 2016; Magnússon et al., 2011; Magnússon et al., 2007; Riesen et al., 2010; Sugiyama et al., 2007), direct observations and inferring of a propagating subglacial pressure wave during the initiation of rapidly rising jökulhlaups (see Einarsson et al., 2017; Einarsson et al., 2016; Jóhannesson, 2002), observation of water volume or emptying of the source lake during jökulhlaups (see e.g. Anderson et al., 2003; Einarsson, 2018; Einarsson et al., 2017; Einarsson et al., 2016; Huss et al., 2007; Werder and Funk, 2009; Werder et al., 2009), observations of subglacial water storage/accumulation (see e.g. Anderson et al., 2003; Einarsson et al., 2017; Einarsson et al., 2016; Huss et al., 2007; Werder and Funk, 2009; Werder et al., 2009), hydro-chemical observations (e.g. Anderson et al., 2003).

In Iceland, some subglacial lakes are fed by meltwater produced by volcanic activity: this is the case, for example, at Grímsvötn which is responsible for one of the most destructive jökulhlaups in the history of Iceland (see e.g. Björnsson, 2002; Björnsson, 2017). Another monitored area, also in the Vatnajökul glacier, is the Eastern and Western Skaftá cauldrons whose subglacial lakes drain in the Skaftá river. Jökulhlaups from the subglacial Skaftá cauldrons flow a distance of about 40 km beneath the outlet glaciers of Tungnaárjökull and then Skaftárjökull to the terminus where the water reaches river Skaftá (see Fig. 1).

Comprehensive reviews of these hydrologic systems, including aspects such as ice surface and bedrock topography, long term mass balance, ice dynamics, hydrology, and other characteristics of jökulhlaups, are presented in several papers, see e.g. Björnsson (1974, 1992, 2002, 2017), Gudmundsson et al. (1995) and Roberts (2005).

A well known mathematical model aimed to explain the observed behaviors has been first presented by Nye (1976) and then refined by other researchers (see e.g. Fowler, 1999; Fowler, 2011; Ng, 1998; Walder and Fowler, 1994). Nye's theory assumes that the drainage occurs through a single subglacial channel, the so-called "R channel" (see Röthlisberger, 1972), that transports water from the ice-dammed lake towards the snout. Two competing processes govern the channel evolution with time, by respectively enlarging and reducing its cross-sectional area. These are the melting of the channel wall by friction and heat transfer from water flow and the creep closure through the action of the excess overburden pressure due to the glacier weight. When they are out of balance, model simulations give a flood behavior similar to some observed jökulhlaups, as, for instance, the 1972 jökulhlaup from Grímsvötn. Nye's insightful and pioneering paper catches probably the main processes controlling the hydrograph shape of the of *slowly* rising jö kulhlaups, namely the conduit-melt-discharge feedback. The model, however, has not been able to explain several recently observed behaviors (see e.g. Einarsson et al., 2017; Einarsson et al., 2016). Indeed, citing Björnsson (2010): "*current models may reconstruct discharge curves while not describing all the factual hydraulic and glaciodynamic processes of each jökulhlaup*". Thus it is now mostly believed that the failure is due to the assumption of drainage through a single channel (see Björnsson, 2010; Einarsson et al., 2017; Einarsson et al., 2016; Johannesson, 2002; Jóhannesson, 2002). Indeed the difference between fast- and slow-rising floods reflects fundamentally different subglacial processes. In the slow-rising case, the discharge grows with time approximately as a power law (see Nye (1976), Eq. (32) at page 193) and the classical theory by Nye is successful in explaining this behavior. On the contrary, in a fast-rising flood, the discharge grows practically linearly with respect to time and this cannot be explained solely by feedback between water flow and conduit enlargement. So, in a fast-rising jökulhlaups it is more suitable to assume a sheet-like flow across large portions of the glacier or underneath, rather than a single channel drainage (see e.g. Flowers et al., 2004).

An important issue concerns the role of temperature. Exit temperatures of the Skaftá jökulhlaups has indeed been measured to be at (or very close to) the melting point (see Einarsson et al., 2017). The same also holds true for slowly rising jökulhlaups from Grímsvötn, which have been measured to be 0.05°C at the outlet (see e.g. Clarke, 2003; Fowler, 2009; Johannesson, 2002). However, a clear physical understanding of the transfer mechanism is still missing and, citing Fowler (2009): "*until*

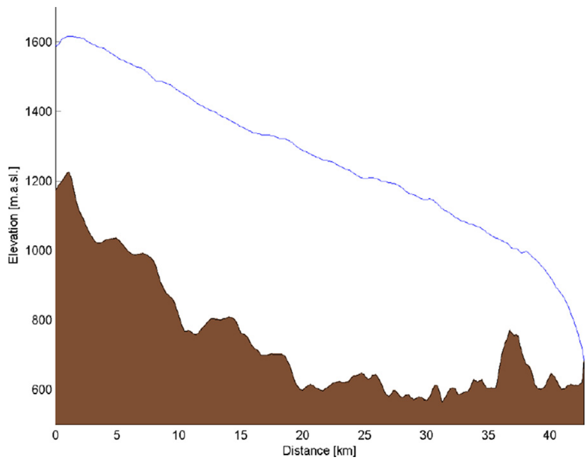

Fig. 2. Longitudinal profile of bedrock and ice surface along jökulhlaup paths from Eastern Skaftá cauldron. Data are provided by the Institute of Earth Sciences at the University of Iceland (from Einarsson (2009a)).

the necessity for the inclusion of heat transfer in the model can be demonstrated by the measurement of positive exit temperatures, it is wiser to neglect the heat advection term in the model ".

Rather few attempts have been made for a physical explanation of fast-rising jökulhlaups. A model to describe fast-rising jökulhlaups has been proposed by Flowers et al. (2004). Such an approach is consistent with the observations of jökulhlaups in Skaftá. In this paper we present an even simpler model with respect to the one in Flowers et al. (2004), purely mechanical too, which may appear rather naïve to experts in glaciology but has the advantage of capturing the main features of fast-rising jö kulhlaups by fitting only three parameters against the flood duration and the discharge. In particular, our model describes those cases in which the drainage occurs through a water sheet which spans everywhere underneath the glacier. We therefore treat the draining layer as a sheet, characterized by a certain permeability, whose width and length are comparable with those of the glacier and whose thickness varies in time. We disregard the thermal effects during the flood and assume that the glacier (which is allowed to move) as well as the bedrock (which does not move) behave as rigid blocks. The glacier lifting (flotation) occurs (and the j ökulhlaups starts) when the hydrostatic pressure due to the lake overcomes the combined action of the glacier weight and resistance (or friction) at the lateral walls of the glacier itself (see Fig. 3).

The outputs of the model are: (i) the discharge which we compare to the experimental data to select the three parameters characterizing the model; (ii) the marginal lake level which we use to validate the model by comparing it with the recorded refilling data. We mainly use data collected by Zóphóníasson (2002); Zóphóníasson and Pálsson (1996), and Einarsson (2009a,b) where several hydrographs of fast-rising jökulhlaups from both the Eastern and Western Skaftá cauldrons are reported.

Other examples of fast-rising jökulhlaups are the one from Grimsvötn in November 1996 (Björnsson (2002)) and the catastrophic one due to Katla, a caldera under the Mýrdalsjökull glacier, in 1918 (see Larsen, 2010; Tómasson, 1996). While Grimsvötn flood of 1996 reached an (estimated) peak of about $4 \times 10^4 \mathrm{m}^3\mathrm{s}^{-1}$, the Katla event was approximately one order of magnitude larger. Both these floods where trigged by a volcanic eruption in the area (see Johannesson, 2002; Jóhannesson, 2002).

The paper develops as follows: in the next Section we present the model for the flood, while the third Section is devoted to the model describing the flood triggering and the almost periodic oscillations of the lake level. In the forth Section we illustrate the fitting procedure and show the simulations referring to various Skaftá events from 1984 to 2008. Our conclusions are drawn in the last Section.

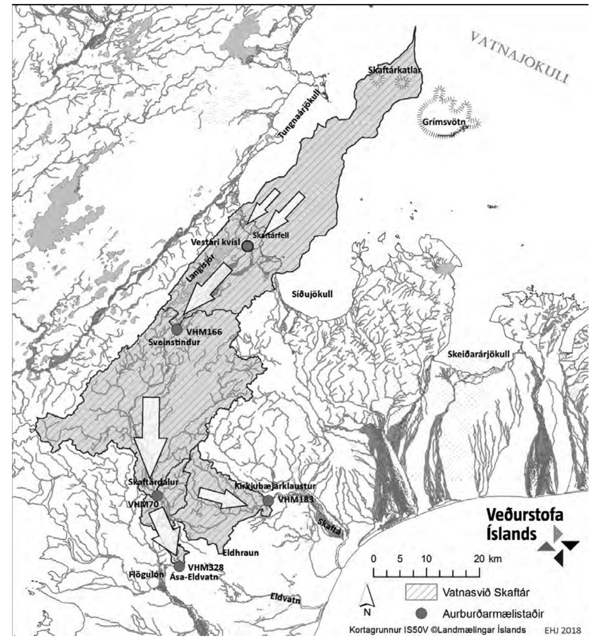

Fig. 3. The Skaftá channel from Vatnajökull to main gauge stations. The size of the arrows indicates graphically varying flow (from Atladóttir (2013); Einarsson et al. (2015), courtesy of Icelandic Meteorological Office).

2. Modelling of the flood event

In this Section we illustrate a model of subglacial-hydrological processes assuming that the subglacial drainage of an ice-dammed lake occurs through a complex system of cross-linked channels that we schematize as a permeable layer, whose effective thickness, denoted as s, varies in time i.e. $s = s(t)$. Fig. 4 shows a schematic representation of the geometry that we consider. In this connection we remark that, although schematic, our figure is not that far from the real situation: indeed the last 15–20 km of the Skaftafellsjökull lies above a typical glacial valley (see, e.g., either Fig 2.1 in Lee (2016) or Fig. 5 in Hannesdóttir et al. (2014)).

The layer length is L and its effective width is a. In particular, we denote by x the longitudinal coordinate so that $x = 0$ is the permeable layer inlet and $x = L$ is the outlet. We denote by $h(t)$ the marginal lake water level with respect to the layer inlet and by h_r the average thickness of the ice "roof" above the lake. The latter is considered as a container with fixed base of area A so that its volume is just $V(t) = A \times h(t)$.

Concerning the discharge through the layer, denoted by Q, we consider

$$Q^2 = \frac{S^n}{\varrho_w g\ K}\left(\frac{\Delta p}{L} + \varrho_w g \sin\vartheta\right), \tag{1}$$

where K is a factor which accounts for the layer permeability of the draining layer, ϱ_w is the water density, g the gravity acceleration, $\Delta p/L$ is the pressure gradient, $S = as$ is the layer cross-sectional area, ϑ is the average slope of the bedrock, considered as a rigid impermeable plane, and n is a characteristic exponent. Eq. (1) is used for flow in open channels and this is not the case here (see the discussion in Fowler (2011), Chapter 11, pp. 773-774). However (1) can also be viewed as a nonlinear Darcy's law for turbulent flow through permeable media as for the so-called Forchheimer law, except that in the latter case the pressure gradient is proportional to the square of the average velocity instead of the square of the discharge (see e.g. Masuoka and Takatsu, 1996). Whether similar laws do exist which are more suitable, is hard to say. The specific choice of (1), essentially is suggested by Nye (1976), and it is motivated only by the requirement of simplicity. About the value of n, possible choices are possible. Here we consider 8/3 in

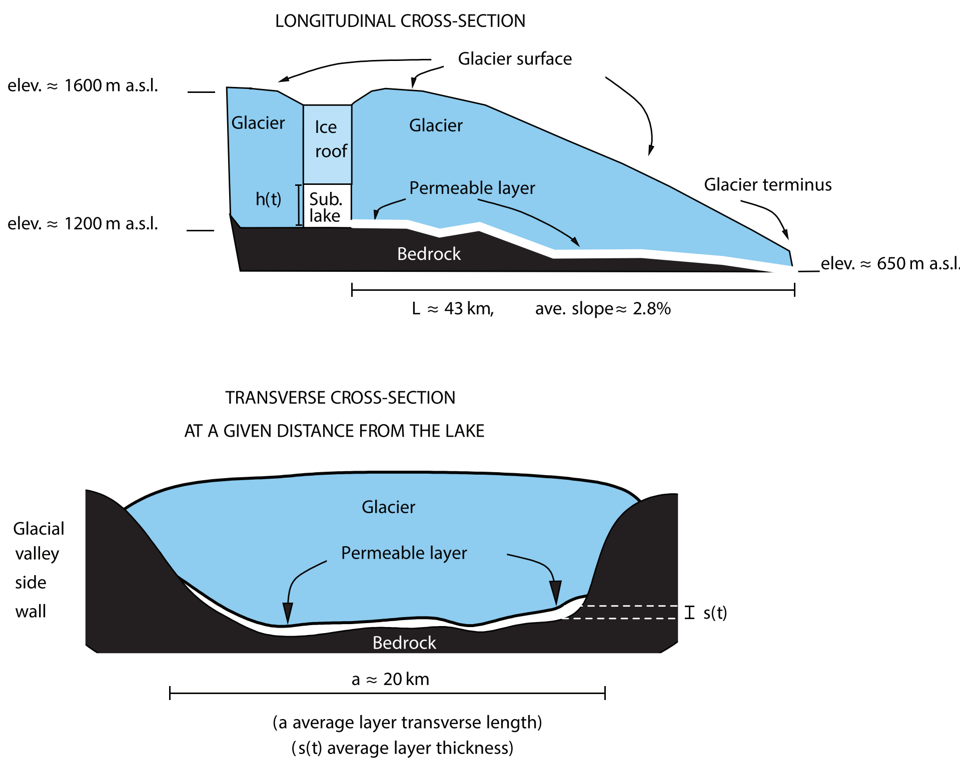

Fig. 4. A schematic representation of a glacier with a subglacial lake. The glacier is considered as a rigid block whose average length and width are L and a, respectively. The glacier thickness varies along the x axis, and we denote it by $H(x)$. The lake is schematized as a container whose effective surface and height are A (fixed) and $h = h(t)$, respectively. The bedrock which sustains the glacier is also supposed to be perfectly rigid and impermeable. The drainage occurs underneath the glacier, through a permeable layer of thickness $s(t)$.

Nye (1976) and 10/3 that would agree with the Manning's law. We have to say, however, that there in not much agreement in literature and other values are possible. In our approach we have chosen the value which gives the best fit of the experimental data.

Concerning the water pressure in the layer, rescaling to zero the atmospheric pressure, we consider a linear profile

$$p = p_{in} - \frac{\Delta p}{L} x, \tag{2}$$

where x is the horizontal axis along the longitudinal section of the glacier, and

$$p_{in} = \varrho_w g \left(h(t) + \frac{\varrho_i}{\varrho_w} h_r \right), \tag{3}$$

and pressure difference

$$\Delta p = p_{in}\left(1 - e^{-s/s_c}\right), \tag{4}$$

with s_c characteristic layer thickness. In the present model, Eq. (4) is physically meaningful only for $s \geq 0$, so that, everywhere in the present paper, the symbol "s" must be interpreted as the *its positive part*, i.e. $s_+ = \max\{s(t), 0\}$. Notice that, when $s \ll s_c$, the pressure field is almost uniform. On the contrary, in case of "a large" opening, $\Delta p/L$ tends to p_{in}/L, i.e. to the one corresponding to a fully developed flow. Thus, recalling (1), the discharge becomes

$$Q^2(t) = \frac{S^n}{L\,K} \left[\left(h(t) + \frac{\varrho_i}{\varrho_w} h_r \right) \left(1 - e^{-s/s_c} \right) + d \right], \tag{5}$$

where $d = L \sin \vartheta$ is the height difference between the draining layer inlet and its outlet.

Remark 1. When the is no hydraulic continuity below the glacier ($s = 0$), the weight of the glacier is sustained by the rigid bedrock. As soon as the pressure due both to the lake and the ice roof has reached a critical value, the layer starts to open. From that moment until the end of the flood ($s > 0$), (4) represents the difference between the overburden pressure due to the glacier and that due to the water flow. For a thin layer, the pressure is more or less uniform in the drainage channel and the flow is predominately driven by gravity. When the opening of the channel grows (and the flow raises considerably), the pressure tends to the linear profile between p_{in}, at $x = 0$, and the atmospheric outlet pressure (rescaled to zero) at $x = L$. The choice of the exponential function as a relaxation effect, is only due, as the exponents in (1), to the aim of designing a mathematical model able to reproduce the observed data.

The aim of the model is to reproduce the discharge time behavior of the fast-rising jökulhlaups by means of (5). To this purpose we have to specify the evolution Eqs for both, $s(t)$ and $h(t)$. Concerning $s(t)$, we write the equation of the vertical motion of the glacier, treated as a rigid block whose average thickness is $\overline{H}$, i.e.

$$\overline{H} = \frac{1}{L} \int_0^L H(x) dx$$

where $H(x)$ is the glacier thickness. Hence, neglecting inertia, we have

$$-La\overline{H}\varrho_i g \cos\vartheta + a \int_0^L p(x,t) dx - \lambda \frac{ds}{dt} = 0, \tag{6}$$

where λ is an effective coefficient accounting for the glacier friction phenomena with the surroundings (or resistance to vertical fracturing of the ice). Notice that the bedrock is supposed rigid as the glacier; therefore (6) is nothing but the one-dimensional Newton law for the unique Lagrangian coordinate s. Evidently, (6) holds true only when s is strictly positive, since otherwise the weight of the glacier is fully supported by the rigid bedrock. There are no specific reasons to assume that the friction term in (6) be proportional to the vertical velocity of the block, except that it is, by far, the simplest choice.

Recalling (2), (3) and (4), Eq. (6) rewrites as

$$\frac{ds}{dt} = \frac{aL\varrho_w g}{\lambda} \left[\left(h + \frac{\varrho_i}{\varrho_w} h_r \right) \frac{1 + e^{-s/s_c}}{2} - \frac{\varrho_i}{\varrho_w} \overline{H} \cos\vartheta \right]. \tag{7}$$

Concerning $h(t)$, water volume conservation entails

$$A \frac{dh}{dt} = -Q + Q_{in}, \tag{8}$$

where Q_{in} is the water input from the lake surroundings (that will be disregarded during the flood event) and Q gives the water volume drained away per the unit time. So, recalling (5), Eq. (8) can be rewritten as

$$\frac{dh}{dt} = -\frac{(as)^{n/2}}{A(LK)^{1/2}}\left[\left(h + \frac{\varrho_i}{\varrho_w}h_r\right)\left(1 - e^{-s/s_c}\right) + d\right]^{1/2} + \frac{Q_{in}}{A}. \tag{9}$$

Remark 2. The assumption of a spatially uniform pressure gradient excludes *a priori* the development of a pressure wave in the subglacial hydraulic system, a possibility considered in Jóhannesson (2002), and more recently by Einarsson et al. (2017). The reason for this choice, as we pointed out in the previous Section, is to maintain the model as simple as possible.

Let us now introduce the dimensionless quantities

$$s^* = \frac{s}{s_{ref}}, \quad h^* = \frac{h}{h_{ref}}, \quad t^* = \frac{t}{t_{ref}}, \quad Q^* = \frac{Q}{Q_{ref}}, \tag{10}$$

where s_{ref}, h_{ref}, t_{ref} and Q_{ref} are reference values. As far as h_{ref} is concerned, we reasonably take[1]

$$h_{ref} = \frac{\varrho_i}{\varrho_w}\overline{H}. \tag{11}$$

For t_{ref} and Q_{ref} we have the field data: we thus take t_{ref} equal to the time duration of the flood event (an easily accessible datum) and Q_{ref} equal to the maximum discharge (other experimental datum).

Omitting the * to keep notation as light as possible, we have

$$\left(\frac{s_{ref}\lambda}{t_{ref}h_{ref}aL\varrho_w g}\right)\frac{ds}{dt} = -\frac{\varrho_i\overline{H}}{\varrho_w h_{ref}}\cos\vartheta + \left(h + \frac{\varrho_i h_r}{\varrho_w h_{ref}}\right)\frac{1 + e^{-s(s_{ref}/s_c)}}{2}, \tag{12}$$

$$\frac{A\left(h_{ref}LK\right)^{1/2}}{t_{ref}\left(a\, s_{ref}\right)^{n/2}}\frac{dh}{dt} = \frac{t_{ref}\,Q_{in}}{Ah_{ref}}t_{ref}\left(a\, s_{ref}\right)^{-n/2} \times \frac{A\left(h_{ref}LK\right)^{1/2}}{t_{ref}\left(a\, s_{ref}\right)^{n/2}} - \sqrt{\left(h + \frac{\varrho_i}{\varrho_w}\frac{h_r}{h_{ref}}\right)\left(1 - \exp\left[-s\frac{s_{ref}}{s_c}\right]\right) + \frac{d}{h_{ref}}}. \tag{13}$$

So, introducing

$$\frac{1}{\alpha} = \frac{\lambda s_{ref}}{aL\varrho_w h_{ref} g t_{ref}}, \quad \beta = \frac{t_{ref}\left(a\, s_{ref}\right)^{n/2}}{A\left(h_{ref}LK\right)^{1/2}} = \sqrt{\frac{h_{ref}\left(as_{ref}\right)^n}{Q_{ref}^2 L\; K}},$$
$$\hat{p}_{out} = \frac{p_{out}}{2\varrho_w g h_{ref}}, \quad \hat{Q}_{in} = \frac{Q_{in}}{Q_{ref}}, \quad \hat{d} = \frac{d}{h_{ref}}, \quad \frac{1}{\gamma} = \frac{s_{ref}}{s_c}. \tag{14}$$

and the new dependent variable

$$z(t) = h(t) + \frac{\varrho_i}{\varrho_w}\frac{h_r}{h_{ref}} = h(t) + \frac{h_r}{\overline{H}}, \tag{15}$$

system (12) and (13) can be rewritten as

$$\begin{cases} \dfrac{1}{\alpha}\dfrac{ds}{dt} = z\dfrac{1 + e^{-s/\gamma}}{2} - \cos\vartheta, \\ \dfrac{dz}{dt} = -\beta\left[z\left(1 - e^{-s/\gamma}\right) + \hat{d}\right]^{1/2} s^{n/2} + \hat{Q}_{in} \end{cases} \tag{16}$$

[1] For instance assuming a triangular shape for $H(x)$, i.e. $H(x) = H(0)(1 - x/L)$, we have $\overline{H} = H(0)/2$ and $h_{ref} = \frac{\varrho_i}{2\varrho_w}H(0)$. In this case, h_{ref} is less than half the height of the ice facing the lake. However, when applying our model to real cases, we will make use of longitudinal profiles of bedrock and ice surface along jökulhlaups paths as those reported in Einarsson et al. (2016) (see Fig. 2): using digitized versions of these profiles it is easy to estimate the mean thickness of the glaciers. For Eastern Skaftá $\overline{H} \approx 470m$. The maximum level of the lake is well below $\overline{H}$.

Table 1
Estimated values of parameters α, β, and γ, from the numerical fit of Eq. (18) against the hydrographs from the Eastern Skaftá cauldron (see Zóphóníasson (2002) and Fig. 3.3 in Einarsson (2009b)).

$n = 8/3$			
Event	α	β	γ
Skaftá 08/1982	73	0.17	5.51
Skaftá 08/1984	33	0.38	15.01
Skaftá 09/1986	50	0.43	9.68
Skaftá 07/1989	76	0.23	5.63
Skaftá 08/1991	57	0.24	6.29
Skaftá 09/2002	60	0.23	5.56
Skaftá 04/2006	109	0.22	5.20
Skaftá 10/2008	104	0.22	5.47
$n = 10/3$			
Event	α	β	γ
Skaftá 08/1982	84	0.13	5.03
Skaftá 08/1984	35	0.35	11.11
Skaftá 09/1986	60	0.33	6.69
Skaftá 07/1989	82	0.20	5.38
Skaftá 08/1991	61	0.21	5.90
Skaftá 09/2002	67	0.20	5.30
Skaftá 04/2006	122	0.19	4.92
Skaftá 10/2008	118	0.19	5.52

Table 2
Values of d, A, $\overline{H}$, h_r, h_0, L, for Eastern Skaftá cauldron. Data are taken from Table 1 of Björnsson (2002).

Parameter	Skaftá
d	550 m
A	4.5 km^2
$\overline{H}$	470 m
h_r	400 m
h_0	100 m
L	40 km

Table 3
Estimated parameter values of $\hat{d}$, z_0 and h_{ref} for Eastern Skaftá cauldron using (11), (14) and (17)

Parameter	Skaftá
$\hat{d}$	1.4
z_0	1.13
h_{ref}	420 m

whose initial conditions are

$$\begin{cases} s(0) = 0, \\ z(0) = z_0 = h_0 + \dfrac{h_r}{\overline{H}}. \end{cases} \tag{17}$$

The discharge (5), rewritten in dimensionless form, is

$$\frac{Q(t)}{Q_{ref}} = \beta\sqrt{s^n(t)\left[z(t)\left(1 - e^{-s/\gamma}\right) + \hat{d}\right]} \tag{18}$$

In model (16), (17) three parameters appear, that is α, β and γ. They are determined fitting (18) against the flood recorded data. Such a procedure is described after the next Section. The initial condition z_0, or better h_0, is the critical level lake at which the emptying process begins. These data are available for Skaftá events (see Table 1 of Björnsson (2002)) and are reported in Table 2.

Remark 3. Steady solutions (z_{st}, s_{st}) of system (16) are all unstable. These solutions can be obtained only numerically. We have calculated them for γ varying in (0,15), $\beta = 0.2$, $\hat{d} = 5$, $\cos\vartheta = 0.99$ and $\hat{Q}_{in} = 2.5 \times 10^{-3}$. We found $(z_{st}, s_{st}) \approx (1, 0.02)$. The value of β is suggested by our simulations (see Table 1). However increasing β of one order of magnitude, z_{st} remains practically the same while s_{st} reduces approximately up to an order of magnitude. Regardless of the value of γ, the eigenvalues are always real with opposite sign, i.e. the steady solutions are saddle points. From the physical point of view this is not surprising since, once a flood event reaches its end, the slowly refilling starts again until the next flood.

Remark 4. By means of Eqs. (14), the two dimensionless parameters α and β can be physically interpreted:

$$\alpha = \frac{\text{weight of the glacier}}{\text{lateral friction force}},$$

$$\beta^2 = \frac{\text{hydro. press. due to the lake}}{\text{hydra. press. due to ice roof + inlet-outlet press.}}.$$

3. Modelling of the flood trigger and lake level oscillations

When the permeable layer almost comes to close, the flood ends and a static condition is reached. Then a new phase begins which may continue for several months or years, during which the refilling process of the lake takes place. Here we try to find a possible, purely mechanical, model which may explain how this static equilibrium breaks and a new flood starts. The leading idea is that, since the bedrock as well as the whole glacier are supposed to be rigid, the only competing forces during the equilibrium phase are the water pressure in the layer (upward), LAp_{in}, with p_{in} given by (3), the glacier weight, $-La\overline{H}\varrho_i g$, and friction lateral force, Φ, between the glacier and the side walls of valley which hosts the glacier. The equilibrium equation is

$$-La\overline{H}\varrho_i g + Lap_{in} + \Phi = 0. \tag{19}$$

The force Φ, exerted on the glacier sides, is due to the interactions between glacier and surroundings. Concerning the second term in (19), we are assuming that there is always a thin subglacial sheet of water ensuring the hydraulic continuity between inlet and glacier terminus.

Physically we expect that there will be a critical value of Φ, denoted by $\Phi_{\max}$, such that if $|\Phi| < \Phi_{\max}$ the glacier is at rest. When the critical load is reached, i.e. when

$$|\Phi| = \Phi_{\max}, \tag{20}$$

a fracture/sliding process occurs on the lateral walls of glacier which starts "lifting". As a result, the draining layer enlarges and the lake emptying begins.

Recalling (3), we rewrite (19) in the following dimensionless form

$$z - 1 + \hat{\Phi} = 0 \quad \Rightarrow \quad \hat{\Phi} = -z + 1, \tag{21}$$

with $\hat{\Phi} = \Phi/\left(La\overline{H}\varrho_i g\right)$. Hence, (20) gives $|-z+1| = \hat{\Phi}_{\max}$, with

$\hat{\Phi}_{\max} = \Phi_{\max}/\left(La\overline{H}\varrho_i g\right)$,

tied to the constraint $z > 1$ (for $z \le 1$ the lake is empty), namely

$$z_0 = \left(\hat{\Phi}_{\max} + 1\right) \quad \Rightarrow \quad h_0 = z_0 - \frac{\varrho_i}{\varrho_w}\frac{h_r}{h_{ref}}. \tag{22}$$

So the "initial" z_0 in $(17)_2$, gives an estimate of the critical load which has to be overcome for triggering the flood.

Concerning the lake oscillations, we introduce t_c as the time at which the draining layer closes. So, just after t_c, the water level in the lake starts again to grow according to the following model

$$\begin{cases} \dfrac{dz}{dt} = Q_{in} \\ z\left(t_c\right) = z_\infty \end{cases} \quad \Rightarrow \quad z(t) = z_\infty + \int_{t_c}^{t} \hat{Q}_{in} dt' \tag{23}$$

where $\hat{Q}_{in}$ the dimensionless recharge rate, has been introduced in (14), and z_∞ is he lake level reached at the end of the emptying process. The new flood occurs after an interval of time T such that $z(T) = z_0$. The phenomenon repeats almost periodically. Thus, assuming a constant in time recharge, the relation between T, lake surface and the recharge rate Q_{in} is

$$\frac{Q_{in}T}{h_{ref}A} = z_0 - z_\infty. \tag{24}$$

Since model (16) allows to estimate z_∞, while z_0 comes from the field data (see Table 3), Eq. (24) can be used to validate the model against the experimental data.

Table 4
Estimated confidence of the fitted hydrographs based on the χ^2−test. Experimental data are due to Zóphóníasson (2002) and Einarsson (2009b).

Event	% of confidence	n	source of the experimental data
Skaftá 08/1982	73	8/3	Zóphóníasson (2002)
Skaftá 08/1984	40	10/3	Zóphóníasson (2002)
Skaftá 09/1986	30	10/3	Zóphóníasson (2002)
Skaftá 07/1989	95	8/3	Zóphóníasson (2002)
Skaftá 08/1991	46	10/3	Zóphóníasson (2002)
Skaftá 09/2002	95	10/3	Zóphóníasson (2002)
Skaftá 04/2006	40	8/3	Einarsson (2009b)
Skaftá 10/2008	40	8/3	Einarsson (2009b)

4. Simulations and comparison with the field data

Solutions $s(t)$, $z(t)$ of system (16) are functions of α, β, and γ. We solve the model with MATLAB®2018b, using ODE45 and the least-squares fitting algorithm implemented by LSQCURVEFIT function. The parameters α, β, and γ are fitted against the available hydrographs Einarsson (2009b); Zóphóníasson (2002): in particular we have considered the events occurred in 08/1982, 08/1984, 09/1986, 07/1989, 08/1991, 09/2002, 04/2006, and 10/2008. The estimated parameters are reported in Table 1.

Remark 5. Table 1 suggests that the choice of a specific value of the exponent n appearing in (1) is rather marginal: the listed parameters do not change very much for the same event. Furthermore, as we will emphasize in a moment, it is probably unrealistic to expect specific value to be valid for all the events, even in the same geographical area.

Simulations require the initial condition z_0 and $\hat{d}$, i.e. h_{ref} which in turn requires $\overline{H}$. The latter depends upon the glacier profile which can be obtained combining GPS measures, radio-echo sounds and are available in several papers (Björnsson (1986, 1988); Björnsson et al. (2000, 1999); Einarsson et al. (2016)). In Table 2 the value of d, $\overline{H}$, h_r, L, and h_0 are reported, while in Table 3 the value of h_{ref}, $\hat{d}$, and z_0 are listed.

Figs. 5, 6, and 7 show comparisons between the simulated hydrographs and the recorded ones from the Eastern Skaftá cauldron, and $z(t)$, i. e. the lake emptying behaviour.

We confined ourselves to show, for the sake of brevity, only three events, namely 07/1989, 08/1991, and 04/2006. However we checked the quality of our results for all the analyzed events by using a standard χ^2-test. The latter requires an estimate of the errors affecting the measures. Unfortunately the hydrographs reported in Zóphóníasson (2002); Zóphóníasson and Pálsson (1996) and Einarsson (2009b) do not show any error bar. However, it should be remarked that hydrographs, especially in case of large events, are known with a non-negligible margin of error. For this reason we decided to assume, prudently, an error percentage of 30%, although, most likely, the error may be larger particularly when the measured discharge is "small". Next, in order to have a statistically significant sample, we extracted from the hydrographs about 15-20 points.

Table 4 displays the confidence degree for each of the eight hydrographs analyzed. The results show an average confidence of $\sim 57\%$ with a peak of 95% for the 1989 and 2002 events. The table shows, for each event, that the best value also depends upon n, the exponent appearing

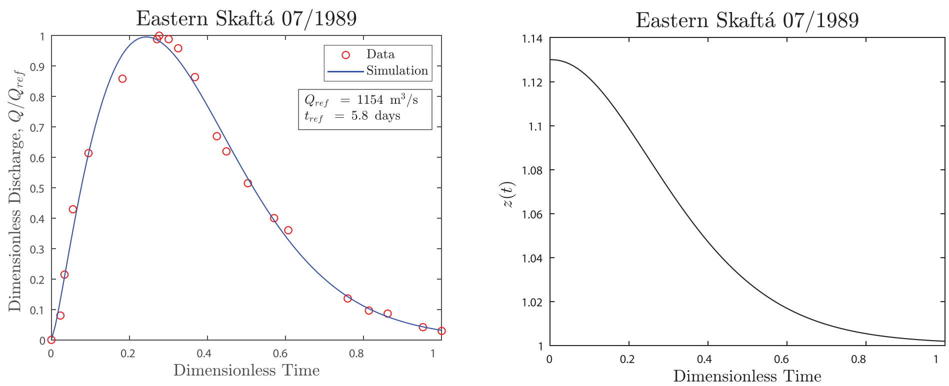

Fig. 5. The 1989 event from Eastern Skaftá: simulated discharge vs. data and simulated lake emptying behaviour. Time scaled to 5.8 days.

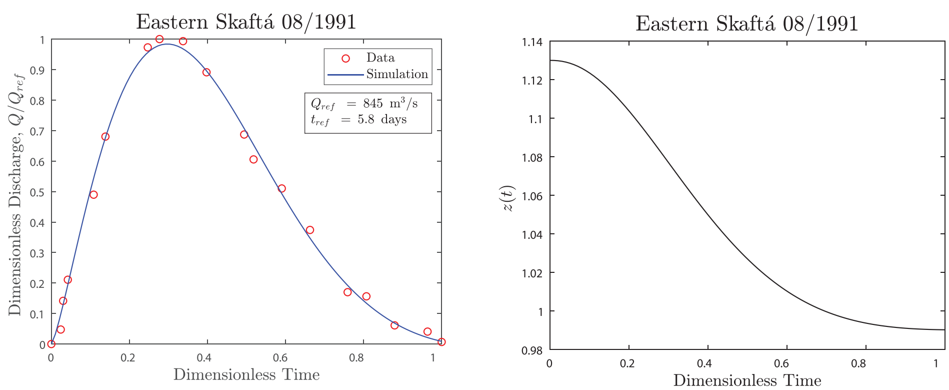

Fig. 6. The 1991 event from Eastern Skaftá: simulated discharge vs. data and simulated emptying lake behaviour. Time scaled to 5.8 days.

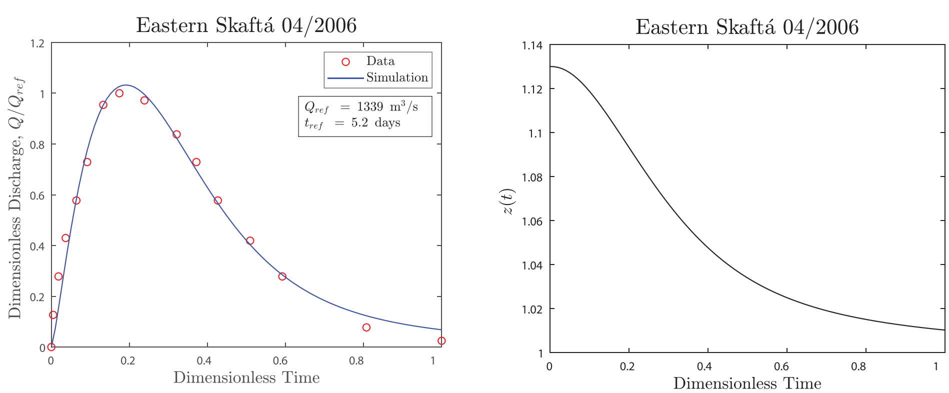

Fig. 7. The 2006 event from Eastern Skaftá: simulated discharge vs. data and simulated emptying lake behaviour. Time scaled to 5.2 days.

in (1). This circumstance is quite reasonable since the mechanical action of each flood may determine non negligible changes of the layer under the glacier, so that, for subsequent events, n may change either. For brevity, we tried only two values (8/3 and 10/3). However, we believe that little modifications of these specific values are equally suitable, due to the lack of precise details about the layer between the base of the glacier and the bedrock.

In Fig. 5, 6, and 7 we have reported the simulated hydrographs and the points extracted from the ones recorded experimentally. The analyzed events are the ones occurred in 1989, 1991, and 2006. The visual agreement is satisfactory as also proved by the χ^2-test results. In particular the model captures very well the "fast" rising of the hydrographs although it does not reproduce with equal accuracy the top and the tale. On the right of each figure, the simulated $z(t)$, i. e. the lake level evolution, is displayed. The level variations, though similar, change from event to event, accordingly to the released water volume. In particular the simulations allow to estimate z_∞, i. e. the level reached by the lake when the flood is over.

Fig. 8 displays $s(t)$ for the eight events analyzed. We note that the maximum value of the dimensionless thickness of the layer is $\mathcal{O}(1)$, which guarantees that s_{ref} is its correct scale. We remark that simulations provide a value of γ of order 10, which, recalling (14), yields $s_c \approx 10\, s_{ref}$. Consequently the exponent in (4) is very small, i.e. the pressure in the layer is practically uniform, suggesting that the flow is mostly driven by gravity as we already anticipated in Remark 1.

The almost periodic refilling model (24) is validated using the recorded data reported in Zóphóníasson (2002) and Einarsson (2009b), as shown in Table 5. Here we have considered five time intervals between subsequent events. The second column reports the measured elapsed time between two subsequent events, the third column the volume of water accumulated in this period and the fourth column the corresponding dimensionless lake raising, which has to be compared with

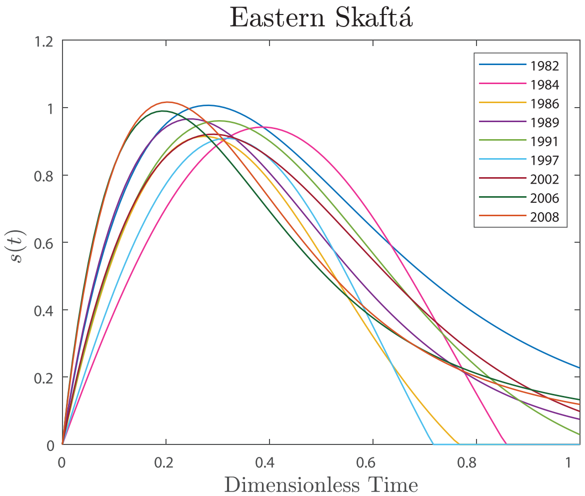

Fig. 8. Evolution of the dimensionless layer thickness for all the Skaftá events considered in the paper.

Table 5
The value of Q_{in}, T are taken from Zóphóníasson (2002) and from table 3.2 in Einarsson (2009b). The value of A is listed in Table 2 while h_{ref} and z_0 are listed in Table 3. Finally z_∞ is the output from simulations.

Subseq. events	T(days)	$Q_{in}T$(Gl)	$\frac{Q_{in}T}{h_{ref}A}$	$z_0 - z_\infty$	% err.
08/1982 – 08/1984	949	336	0.18	0.12	33
08/1984 – 09/1986	826	239	0.13	0.21	17
09/1986 – 07/1989	957	279	0.15	0.18	23
07/1989 – 08/1991	747	219	0.12	0.13	13
04/2006 – 10/2008	908	295	0.16	0.12	17

the one predicted by the model (fifth column). The difference, given in percentage, between the experimental data and the theoretical ones is reported in the last column.

5. Conclusions

The aim of the paper is to present a simple model for explaining fast-rising jökulhlaups. Clearly, because of its simplicity, our approach does not consider some intriguing problems as the type of channelization of the drainage system, the wall erosion, the sediment dynamics, the pressure wave dynamics and so on. Moreover, according to suggestions that come from previous studies in this area, we disregard all effects of heat transfer from flowing water to the glacier. Thus, citing Flower (see Flowers et al. (2004), page 4), *limitations are inherent in our rudimentary approach where we have omitted some processes and simplified others, yet this framework provides a hopeful starting point for understanding other floods that have eluded classical jökulhlaup theory*.

In the case of fast-rising floods, this hypothesis seems acceptable to a reasonable extent. We also treat the glacier as a rigid block and assume that the drainage occurs through a permeable layer underneath the glacier. The flow through the layer is governed by a Forchheimer-type law, assumed by all authors after Nye. The competing mechanisms, once the flood has started, are the weight of the glacier, the water pressure due to the marginal-lake level and the resisting force at the glacier side walls. The maximum value of the shearing stress at the side walls provides the maximum lake level that must be reached between two subsequent floods. Our model is able to reproduce the hydrographs of eight well known fast-rising jökulhlaups in Iceland from the Eastern Skaftá cauldron from 1982 to 2008. This conclusion appears to be confirmed by the good confidence values provided by the χ^2-test: although some of them are well below 50%, we have an average percentage of about 57%, which considering the great uncertainty affecting the field data, can be considered a quite acceptable result. Floods that are subsequent in time are well simulated in the sense that the lake level variation agree with data reported in Einarsson (2009b); Zóphóníasson (2002). The percentage difference between the model outcome $z_0 - z_\infty$ and the experimental data $Q_{in}T/(h_{ref}A)$ remains below 30% except for the pair 1982-1984 (which is about 33%). It should be considered, however, that some experimental values, as A, h_0 and h_r, have a significant degree of uncertainty. Nevertheless, we believe that, although naïve, the usefulness of our approach is that it is able to catch the main aspects of this complex phenomenon, without involving several parameters which are very difficult to estimate.

Declaration of Competing Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

We would like to thank Dr. S. Zóphóníasson and Dr. B. Einarsson for providing all hydrographs and some useful suggestions.

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