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## Modeling the evolution of the Antarctic ice sheet in the last 200 ky

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### Abstract

The evolution of the Antarctic ice sheet for the last 200 ky is simulated with a finite difference thermomechanical model based on the shallow ice approximation. The model depends on the surface temperature, the ice accumulation rate, the geothermal heat flux and the basal sliding coefficient, which are estimated with large uncertainty. Therefore a sensitivity analysis is applied to assess the most critical parameters.

*Keywords:* glaciology; thermomechanical modelling; shallow ice approximation; Antarctic ice sheet; Hurst exponent.

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### 1. Main text

The presence of water at the base of the Antarctic ice sheet has been revealed from several geophysical surveys and requires that the temperature at the bedrock is close to the melting point. The melt rate is one of the terms of the mass balance of the Antarctic ice sheet, which is a huge reservoir of (frozen) fresh water, and therefore is an important parameter to assess the impacts of climate change on the Earth. Since environmental conditions prevent from acquiring direct information, the knowledge about the physical processes occurring in the Antarctic ice sheet is still incomplete. Simulation models can be used to test different hypotheses and to plan field or remote sensing surveys. As an example a dynamical model of ice sheets is presented here: it has been applied to simulate the evolution of the Antarctic ice sheet during the last 200 ky. The uncertainty of some model outcomes due to the uncertainty of some input parameters is evaluated with a sensitivity analysis.

#### 1.1. The model

The basic equations (mass, momentum and energy balance) are coupled with each other. In fact, the heat equation includes convection and heating due to ice strain, whereas the ice flow depends on temperature, because the relationship between the strain rate and the deviatoric stress tensor (Glen's law) non linearly depends on ice temperature. Some assumptions (homogeneous and cold ice, quasi-static and hydrostatic conditions, SIA – Shallow Ice Approximation) are introduced to reduce the complexity of this non-linear thermomechanically-coupled model.

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The equations have been discretised with a finite difference scheme: a staggered grid and an up-wind scheme for the convective terms are used; an explicit scheme has been applied for the mass conservation equation, while the heat equation is discretised with an implicit scheme for the vertical heat conduction and an explicit scheme for the convective and source terms. The boundary conditions depend on the surface temperature, the surface accumulation rate, the geothermal heat flux and the basal sliding coefficient.

The model has been implemented with an original computer code and has been validated by application to synthetic case studies taken from the EISMINT experiments (Huybrechts et al., 1996; Payne et al., 2000).

### 1.2. The application and the sensitivity analysis

The surface temperature of the Antarctic ice sheet for the last 200 ky is taken from Huybrechts (1990) and depends on  $T_{s0}$ , the estimate of the present day surface temperature field, and on  $\beta_T$ , the vertical gradient of the surface temperature. The accumulation rate is taken from Huybrechts et al. (2009) and depends both on the surface temperature and on  $M_{s0}$ , the estimate of the present day accumulation rate. The geothermal heat flux,  $G$ , is assumed to be constant in time, but varying in space from 51  $\text{mWm}^{-2}$  for Eastern Antarctica to 69  $\text{mWm}^{-2}$  for Western Antarctica (Llubes et al., 2006).

The parameters  $T_{s0}$ ,  $\beta_T$ ,  $M_{s0}$  and  $G$  and the coefficient of basal sliding,  $B_s$ , influence the model outcomes of principal interest, i.e. the ice volume ( $V_{ice}$ ), the wet area at the ice sheet base ( $S_{wet}$ ), the total outgoing flux ( $\Phi_{out}$ ), the total basal melt rate ( $BMR$ ). Therefore the prediction scaled sensitivity (Hill and Tiedeman, 2006) is used for the sensitivity analysis:

$$pss = \frac{dX}{X_{ref}} \cdot \frac{Y_{ref}}{dY} \quad (1)$$

where  $pss$  represents the relative variation of the predicted quantity  $X$  corresponding to a unit relative variation of a model parameter  $Y$ .

Two kinds of predicted quantities have been used for the sensitivity analysis. First, the predicted quantities have been considered as the time averages of  $V_{ice}$ ,  $S_{wet}$ ,  $\Phi_{out}$ ,  $BMR$ . Then, it was observed that  $S_{wet}$  and  $\Phi_{out}$  show a pseudo-chaotic behavior, which reflects in a power spectrum  $P(\nu) \propto \nu^m$  for a range of frequencies  $5 \cdot 10^{-5} \text{ Hz} < \nu < 5 \cdot 10^{-3} \text{ Hz}$ . For  $S_{wet}$  and  $\Phi_{out}$ ,  $m \approx -2$ , which corresponds to a Hurst exponent of 1/2 (Turcotte, 1999): the spectral Hurst exponent, a measure of the chaoticity of a dynamical system, was estimated and was also considered as a predicted quantity.

The results both for the time-averaged model outcomes and for the analysis of the Hurst exponent show that  $T_{s0}$  is the parameter for which the  $pss$  are greatest and the presence of subglacial water, estimated through the area of the wet surface, is also noticeably influenced by the geothermal heat flux.

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