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The multi-vehicle profitable pick up and delivery routing problem with uncertain travel times

Maria Elena Bruni[a,*], Dau Quoc Toan[b], Le Hoang Nam[b]

[a]*Department of Mechanical, Energy and Management Engineering, Unical, Rende (CS) 87036, Italy*
[b]*Department of Mathematics and Computer Science, Unical, Rende (CS) 87036, Italy*

Abstract

This paper addresses a variant of the known selective pickup and delivery problem with time windows. In this problem, a fleet composed of several vehicles with a given capacity should satisfy a set of customers requests consisting in transporting goods from a supplier (pickup location) to a customer (delivery location). The selective aspect consists in choosing the customers to be served on the basis of the profit collected for the service. Motivated by urban settings, wherein road congestion is an important issue, in this paper, we address the profitable pickup and delivery problem with time windows with uncertain travel times. The problem under this assumption, becomes much more involved. The goal is to find the solution that maximizes the net profit, expressed as the difference between the collected revenue, the route cost and the cost associated to the violation the time windows. This study introduces the problem and develops a solution approach to solve it. Very preliminary tests are performed in order to show the efficiency of developed method to cope with the problem at hand.

Keywords: Pick up and delvery; Profits; Soft time windows;

1. Introduction

The Pickup and Delivery Problem (PDP) is one of the most studied problem in the routing literature. It is a variant of the vehicle routing problem that uses a homogeneous capacitated and limited fleet of vehicles, parked at a single depot, in order to serve the customers' pickup and delivery requests. Over the last decades, researchers have studied many variants of the PDP and they have used various algorithms to solve those variants (Berbeglia et al. (2009); Dumitresci et al. (2010); Hernandez-Perez et al. (2007)). Very often, time windows are included in the considered problem, leading to the so-called Pickup and Delivery Problem with Time Windows (PDPTW).

In this area, branch-price-and-cut exact approaches have been shown to be able to provide state-of-the-art results (Ropke and Cordeau (2009); Baldacci et al. (2011); Parragh e tal. (2000)). Metaheuristic methods have been suc-

* Corresponding author. Tel.: +39-494038
E-mail address: mariaelena.bruni@unical.it

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cessfully applied to efficiently solve larger instances Ropke and Pisinger (2006); Nalepa and Blocho (2016); Nguyen et al. (2015).

When the constraint of visiting all the customers is relaxed, the problem becomes selective. This variant has received scant attention from the literature so far. For the single vehicle case, Gribkovskaia and Laporte (2008) applied tabu search to the PDP with selective pickups arising in the routing of supply vessels through offshore installations. A branch-and-cut algorithm was later proposed by Gutierrez et al. (2009, 2010). A memetic algorithm was presented in Ting and Liao (2013). The multi-vehicle profitable PDP has been studied in Qiu et al. (2014, 2017); Gansterer et al. (2017). Prive et al. (2006) developed a heuristic for a practical problem involving the delivery of soft-drinks and the collection of empty cans and bottles. A variant of the problem with time windows, profits and reserved customers, has been introduced in Li e t al. (2016). There, the visit of the reserved customers is mandatory while optional customers may be visited by other carriers or not visited at all. A profit is associated to each customer, and the aim is to build, for each carrier, a feasible route including all mandatory customers and possibly some optional customers, in order to maximize the difference between the profit collected from the visited customers and the transportation cost. The problem addressed in this study is structurally similar to the problem presented in Li e t al. (2016), since serving all the requests is not mandatory and a profit is associated with each request. Our problem involves an additional realistic feature into the decision-making process. Instead of ignoring the uncertainties associated with the problem parameters-which may lead to near-optimal routing plans or even solutions which actually turn out to be infeasible after the uncertainty disclosure-we explicitly consider consider travel times uncertain, and notably, represented by continuous random variables at the planning level. In particular, we deal with the stochasticity of arrival times of the vehicles at customer locations, which are function of the random travel times, and hence, are them-self random variables. This challenges the compliance to time windows restrictions, especially considering that vehicles often operate in traffic congested cities. Since in real-life contexts, a full compliance may become quite costly at the planning level, we allow time windows violations at the cost of a penalty.

It is worth mentioning that a variety of operational planning problems in transportation logistics have been very recently studied in the literature, assuming continuous distributions for the uncertain travel times. The interested readers are referred to Bruni et al. (2020a,b,c); Tas e tal. (2014). The paper is structured as follows. In the next section, we provide the problem description and the mathematical formulation. Section 3 provides an exact solution method to solve the problem. Computational results are presented in Section 4. The final section contains conclusions and directions for future research.

2. Problem description

We consider the design of vehicle routes for a set of customers who specify transportation requests from origin (pickup) to destination (delivery) points. Users typically impose time windows with respect to these locations. In this study, we assume that all orders are known before the operations start. This assumption is reasonable in many online shops that receive their orders before they build delivery routes. The available known requests shall be served by a fleet K of homogeneous capacitated vehicles (with a maximum capacity Q) that can consolidate different requests in the same trip as long as their loads fit into the vehicle capacity. Nodes can be visited at any order provided that the same vehicle visits both the pickup and delivery nodes of a request and that the pickup node is visited earlier (but not not necessarily immediately before) than the delivery node (precedence relation). The problem is selective, since it is not mandatory to deliver all the requests. The profitability is evaluated as the net profit expressed as the revenue π_i collected from the requests $i \in P$ minus the traveling cost and the cost related to the possible violation of the time windows constraints. We now formally define the mathematical model, by introducing the necessary notation hereafter. Each request is characterized by a pair of stops $(i, n+i)$, $i \in P$ and $(n+i) \in D$, where $P = \{1, ..., n\}$ represents the set of pickup nodes and $D = \{n + 1, ..., 2n\}$ the set of delivery nodes. For each node $i \in V = P \cup D$, q_i represents the quantity loaded or unloaded and $[e_i, l_i]$ the times (early, late) delimiting the requested time window. Let c_t (c_e) the unitary cost associated to the tardiness (earliness), evaluated with respect to the time windows. The problem may be defined on a complete directed graph $G = (N, A)$ where $G = \{0, 0^* \cup N\}$ (here 0 and 0^* are two copies of the depot. We then introduce a binary decision variable x_{ij}^k that equals 1 if vehicle k travels from node i to node j, incurring a routing cost c^k_{ij}. Each arc $(i, j) \in A$ has an associated travel time t_{ij}. The deterministic problem can be formulated as:

$$Max \sum_{k\in K}\Big(\sum_{i\in P}\sum_{j\in V} x^k_{ij}\pi_i\Big) - \sum_{k\in K, i\in V, j\in V} c^k_{ij}x^k_{ij} - c_t \sum_{k\in K, i\in V} [u^k_j - l_j]^+ - c_e \sum_{k\in K, i\in V} [e_j - u^k_j]^+ \quad (1)$$

$$\sum_{k\in K}\sum_{j\in V} x^k_{ij} \le 1 \quad i \in P \quad (2)$$

$$\sum_{j\in V} x^k_{ij} - \sum_{j\in V} x^k_{(n+i)j} = 0 \quad i \in P,\ k \in K \quad (3)$$

$$\sum_{i\in V} x^k_{0i} = \sum_{i\in V} x^k_{i0^*} = 1 \quad k \in K \quad (4)$$

$$\sum_{j\in V} x^k_{ji} - \sum_{j\in V} x^k_{ij} = 0 \; i \in V,\ k \in K \quad (5)$$

$$u^k_j \ge u^k_i + t_{ij} - B^k_{ij}(1 - x^k_{ij}) \quad i,\ j \in V,\ k \in K \quad (6)$$

$$w^k_j \ge w^k_i + q_j - W^k_{ij}(1 - x^k_{ij}) \quad i \in V,\ j \in V,\ k \in K \quad (7)$$

$$max\{0, q_i\} \le w^k_i \le min\,\{Q, Q + q_i\} \; i \in V,\ k \in K \quad (8)$$

$$x^k_{ij} \in \{0, 1\} \; i, j \in V,\ k \in K \quad (9)$$

$$u^k_i \ge 0 \; i \in V, k \in K \quad (10)$$

The objective function (1) is a composite form that maximizes the difference between the revenue, the route cost, as well as the cost associated with time windows violations, expressed in terms of tardiness and earliness. Constraints (2) and (3) ensure that each request is served at most once and, if it is served, its pickup and delivery nodes are visited by the same vehicle, respectively. Constraints (4) guarantee that the route of each vehicle starts and ends at the depot. Constraints (5) are flow conservations constraints. Inequalities (6) define the time at which each vehicle $k \in K$ begins the service at node $j \in V$. This time is defined as the sum of the arrival time at the preceding node i in the route u^k_i plus the traveling time of the segment (i, j). This constraint should be enforced only if the segment (i, j) belongs to the route of the vehicle k and therefore is expressed as a big-M constraint, where $B^k_{ij_s}$ is a big constant that can be set to $B^k_{ij} = \max\{0, l_i + t_{ij} - e_j\}$. Constraints (7) define the vehicles load at each node j. Similarly to the preceding case, the big-M constraints are formulated through the use of the constant $W^k_{ij} = min\{Q, Q + q_i\}$ in constraints (8) in order to impose vehicles capacity restrictions. Constraints (9) and (10) define the nature of the variables.

When the travel times are deterministic, this mixed integer problem defines a NP-hard problem, which can be efficiently solved by means of tailored solution approaches (Riedler and Raidl (2018)). If instead uncertainty is considered, the problem becomes much more involved and the mathematical model almost useless. In fact, since the travel times are random variables, also the arrival times are random variables (hereafter denoted by $\tilde{u}_i$, $\forall i \in V$) and, as such, the tardiness and the earliness. A common approach under uncertainty is to consider a risk neutral viewpoint of the decision maker, notably implemented through the minimization of the expected value. In our case, for each node i the expected tardiness can be defined as follows:

$$\mathbb{E}[T_i] = \mathbb{E}[\tilde{u}_i|\tilde{u}_i \ge l_i],$$

Analogously, the expected earliness is defined as:

$$\mathbb{E}[E_i] = \mathbb{E}[\tilde{u}_i|\tilde{u}_i \le e_i].$$

The evaluation of these quantities, is in general not trivial, even for a given solution (route), and depending on the particular distribution function used to model the travel times, can be a straightforward or a very complicates task. In fact, we note that $\tilde{u}^k_{i+1}$ depends on $\tilde{u}^k_i$ and on the random travel time $\tilde{t}_{i\,i+1}$ between i and $i+1$, since we can rewrite $\tilde{u}^k_{i+1} = \tilde{u}^k_i + \tilde{t}_{i\,i+1}$. Hence, the arrival time at each node is the sum of the travel times associated to the links belonging to the route (r^k_i) connecting the depot to the node i. Hereafter, we assume that the travel times are independent random variables. Although this hypothesis is barely satisfied, especially in urban contexts, it allows us to explicitly evaluate the objective function of the stochastic counterpart of the problem (1)–(10). In fact, the calculation of the sum of independent random variables is the convolution of their distribution and it can be quite complex based on the probability distributions of the random variables involved and their relationships. For a family of distributions closed under convolution, the task becomes easier, since the sum of random variables has the same distribution of the original variables. Within this family, the Gamma probability distribution has been used to model travel times in routing problems, since it is a good one to use for any skewed distribution.

In the next section, assuming that travel times are distributed as Gamma random variables, we will present a solution method to solve the problem defined above.

3. Solution approach

A random variable Y is said to have a Gamma distribution if and only if its density function is

$$f(Y) = \frac{Y^{\alpha-1}e^{-Y/\beta}}{\beta^{\alpha}\Gamma(\alpha)} 0 < Y < \infty. \tag{11}$$

Where α and β are both greater than zero and are called the shape and the scale parameters, respectively. The gamma function $\Gamma(\alpha)$ is defined by the integral: $\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}dy$. The cumulative distribution function is denoted by $F_{\alpha,\beta}(\hat{y})$ and is defined as $P(Y \leq \hat{y}) = \int_{-\infty}^{\hat{y}} f(y)dy$.

If we assume that the random travel time of the link (i, j) $\tilde{t}_{ij}$ is Gamma distributed with parameters αd_{ij} and β , then the arrival time at node i (denoted, for the sake of brevity by Y), is again a Gamma variable with shape $\alpha' = \alpha \sum_{(i,j)\in r_i^k} d_{ij}$ scale parameter $\beta' = \beta$. Hence, the expected tardiness can be defined Tas e tal. (2014) as

$$\mathbb{E}[T_i] = \int_{l_i}^{\infty} (y - l_i)f(y)dy = \int_{l_i}^{\infty} (y - l_i)\frac{y^{\alpha'-1}e^{-y/\beta'}}{\beta'^{\alpha'}\Gamma(\alpha')}dy = \int_{l_i}^{\infty} y\frac{y^{\alpha'-1}e^{-y/\beta'}}{\beta'^{\alpha'}\Gamma(\alpha')}dy - l_i\int_{l_i}^{\infty} \frac{y^{\alpha'-1}e^{-y/\beta'}}{\beta'^{\alpha'}\Gamma(\alpha')}dy =$$

$$= \beta'\alpha'(1 - F_{\alpha'+1,\beta'}(l_i)) - l_i(1 - F_{\alpha',\beta'}(l_i)). \tag{12}$$

The expected delay can be evaluated analogously. It is beneficial to recall that this evaluation is only possible for a given solution of the problem, i.e. for a given set of routes, where the sequence of the arcs in each route is known. Only in this case, in fact, the distribution function of the arrival times can be properly defined as the sum of travel times on arcs traversed along its route by the vehicle until that node.

Since a route r^k is a sequence of nodes, a graph search could help to enumerate all feasible permutations that satisfy the constraints. The algorithm assigns states to each node: each state associated with node i represents a route from the depot to i, and has an associated objective function. The algorithm repeatedly extends each state to generate new states. The extension of a state corresponds to appending an additional arc (i, j) to the route. This operation is repeated until all states have been extended in all feasible ways. In order to ensure feasibility, two unsorted sets (UP and UD) are used for the unvisited pickups and delivery requests, respectively while an ordered list R stores the current route. The route expansion is performed within a recursive procedure that carries out an exhaustive search of the routing space in a scalable manner. In particular, a depth-first-search algorithm is implemented in a recursive way to implement the above operations. The search tree is structured into levels. At the root of the search tree (level 0), there is only the empty path and the vehicles are at the depot. When branching a node, a partial path is expanded in every possible feasible way, meaning that the routes do not contain sub-tours, while the one-to-one correspondence of pickup nodes and delivery nodes and the capacity restrictions are maintained. At each node, the following options are possible.

- Pick up a new request i and remove accordingly the request from the set UP. Since the future delivery of the requests must be done, the node $i + n$ is inserted into UD. The route is updated accordingly.
- Deliver a request j and remove it from UD, updating the route.

If the new requests make the total load exceed the capacity limit, the recursion is stopped. The route expansion is also subject to the condition that the newly created route will have an upper bound lower than the current best objective function value. Accordingly, our implementation prunes inferior routes, where the expected potential profit (evaluated as $\sum_{k\in K}\sum_{i\in r^k}(\pi_i - \sum_{j\in r^k} c^k_{ij} - c_t\mathbb{E}[T_i] - c_e\mathbb{E}[E_i])$ remains below the current best objective function OF_{best}, evaluating the theoretical profit from handling all remaining requests that are still available. If this theoretical value remains below the current best solution the recursion is aborted.

The core of the method is the recursive function traversing the tree, which is reported in Algorithm 1. The function has four parameters, namely the set of requests UP, the set of requests that were picked up and need to be delivered UD, the routes R and the number of vehicles k processed so far. The algorithm is called at the beginning with the set UP initialized with all the pickup requests, $UD = \emptyset$ and $k = |K|$. The OF_{best} stores the optimal objective function value.

Algorithm 1 GraphSearch (UP, R, UD, k)

```
if k=0 then
  return because all the vehicles routes have been processed
else if vehicle capacity is violated then
  return because the route is infeasible
end if
Compute the objective function of the current route plan R and add the theoretical profit from handling all remaining
requests that are still available.
if the computed objective function (OF_current) remains below the current-best current-best (OF_best) then
  abort the recursion here.
end if
for i ∈ UP ∪ R do
  if i ∈ UP then
    UP ← UP \ {i}; UD ← UD ∪ {i + n}; R ← R ∪ {i} .
    Call GraphSearch ( UP, R, UD, k.)
    UP ← UP ∪ {i}; UD ← UD \ {i + n}; R ← R \ {i}.
  else
    UD ← UD \ {i}; R ← R ∪ {i}
    Call GraphSearch ( UP , R, UD , k.)
    UD ← UD ∪ {i}; R ← R \ {i}
  end if
  if UD = ∅ then
    Compute the objective function of the current route plan R
    if it is greater then OF_best then
      update the best route and current optimum
    end if
    Add 2n + 1 and 0 to R
    Call GraphSearch (UP , R, UD , k − 1)
    Remove 2n + 1 and 0 from R
  end if
end for
```

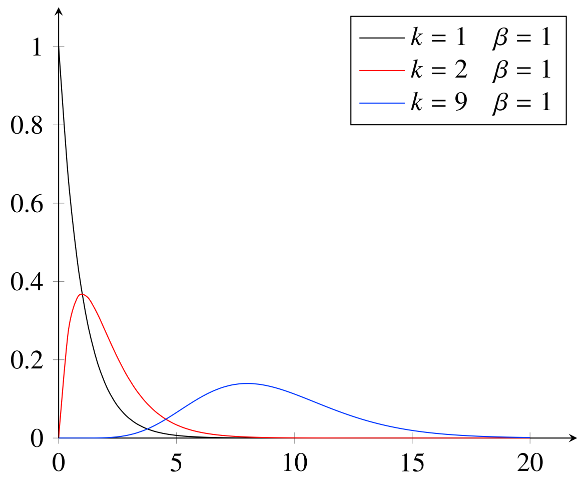

Fig. 1. Shape of the Gamma distribution for different parameters.

4. Computational experiments

The algorithm has been implemented in Kotlin language, an enhanced Java language, and executed on an Intel Core i7-7700HQ at 2.80GHz and 16 GB RAM. The test problems were derived from the instances used in (Gansterer et al. (2017)). In the data instances customers are scattered on a two-dimensional plane. The number of requests we consider is 20 and 50, with a number of vehicles of two and three, respectively. Following the original data set we have considered:

- the same revenue for all the locations (denoted by F in te instance)
- a revenue which depends on the demand (denoted by P in the instance)
- a randomly generated revenue (denoted by R in te instance)
- small vehicles capacity (denoted by S in te instance)
- large vehicles capacity (denoted by L in the instance)

Moreover, we have considered wide time windows and regular time windows. The cost parameters were set as follows $c_{ij} = d(i, j)$, $c_e = 0.1$, $c_t = 0.3$. The shape and scale parameters in the Gamma distribution for each arc (i, j) were set to $\alpha = d_{ij}$ and $\beta = 1$. As the name implies, changing the value of α will change the shape of the distribution. Hereafter, we report the plot of the density function for three different values of α. As evident, we are able to model different distributions, from very skewed to almost normal ones. We mention that a comparison of the proposed algorithm with the deterministic approach (or any other state-of-the art algorithm for the deterministic problem) is not interesting, given the completely different nature of the corresponding problems. Moreover, assuming worst-case travel times and solving the deterministic problem is not informative, since the worst-case solution clearly overestimates the expected tardiness.

Table 1. 20 nodes instances

	Regular TW		Large TW	
	OF	*CPU*	*OF*	*CPU*
FS	36186.56	0.082	37705.62	0.126
FL	34433.60	6.624	37373.77	0.878
PS	51186.56	0.082	52705.62	0.126
PL	41833.60	6.780	44773.77	0.770
RS	44108.56	0.085	45627.62	0.126
RL	40240.60	6.763	43180.77	0.874

Table 2. 50 nodes instances

	Regular TW		Large TW	
	OF	*CPU*	*OF*	*CPU*
FS	45378.26	0.146	50335.41	1.467
FL	43541.51	1329.33	51622.68	56.962
PS	120278.3	0.146	137335.4	1.465
PL	126941.5	1345.45	135022.7	57.050
RS	68491.26	0.145	75712.41	1.446
RL	63551.51	1301.15	76243.68	56.988

Table 1 reports the results obtained for 20 nodes, in terms of objective function value (columns with heading OF) and CPU time in seconds (columns with heading CPU). As evident, the algorithm is able to find the optimal solution in a very limited amount of time and always within 7 seconds. On average the CPU time is around 3.5 second for regular time windows and less the 1 second for large time windows. The larger the time windows are, the more are the feasible solutions. The percentage deviation between the objective function value of the problems with regular and large time windows is around 5.5%. In particular, a decrease in the time windows range would lead to a decrease in the profit, related to the growth of the expected cost of the tardiness and earliness. In Table 2, the results for the 50 nodes instances are reported. We observe that for the instances where long tours are feasible (i.e. vehicles have large capacity), the solution time considerably increases. This is due to the time needed to search through feasible routes, which in this case are quite a high number. We also notice that for the other cases, the solution time is still limited to a few seconds.

5. Conclusions

Travel time uncertainty has an impact to congestion of road networks. In this paper, we dealt with this issue within a pick up and delivery routing problem with time windows. Moreover, we present a variant of the problem, which applies the option to select customer requests on the basis of a profit. This combination of features is of high practical relevance in particular in last-mile distribution systems. By adopting a risk-neutral perspective, and under the assumption that travel times are independent Gamma distributed random variables, we show that the expected earliness and tardiness costs can be evaluated in a closed form. As solution method we propose a graph search approach. Our experiments show that we can obtain exact solutions in a reasonable amount of time. To extend the current research, we plan to implement suitable heuristic methods for larger instances. In some cases, businesses would also like to consider a risk-adjusted costs. Research along this line is still far to be mature and deserves further attention.

References

Baldacci, R., Bartolini, E., Mingozzi, A. (2011). An exact algorithm for the pickup and delivery problem with time windows. Oper. Res. 59 (2), 414–426 .

Beraldi, P., Bruni, M. E., Lagan‘a, D., and Musmanno, R. (2019). The risk-averse traveling repairman problem with profits. Soft Computing, 23(9), 2979-2993.

Berbeglia, G., Cordeau, JF., Gribkovskaia, I. , Laporte, G.(2007). Static pickup and delivery problems: a classification scheme and survey. Top 15(1), 1–31. https://doi.org/10.1007/s11750-007-0009-0

Bruni, M.E., Khodaparasti, S., Beraldi, P. (2020). The selective minimum latency problem under travel time variability: An application to post-disaster assessment operations. Omega, 92, 102154.

Bruni, M.E., Beraldi, P., Khodaparasti, S. (2020). A hybrid reactive GRASP heuristic for the risk-averse k-traveling repairman problem with profits. Computers and Operations Research, 115, 104854.

Bruni, M.E., Khodaparasti, S., Nucamendi-Guilln, S. (2020). The bi-objective minimum latency problem with profit collection and uncertain travel times. ICORES 2020 - Proceedings of the 9th International Conference on Operations Research and Enterprise Systems pp. 109-118.

Dumitrescu, I., Ropke, S., Cordeau, J.-F., Laporte, G. (2010). The traveling sales- man problem with pickup and delivery: polyhedral results and a branch- and-cut algorithm. Mathematical Programming, 121 (2), 269–305.

Gansterer, M. , Kucuktepe, M. , Hartl, R.F.(2017). The multi-vehicle profitable pickup and delivery problem. OR Spectr. 39 (1), 303-319.

Gribkovskaia, I., Laporte, G., (2008). One-to-many-to-one single vehicle pickup and delivery problems. In: Golden, B.L., Raghavan, S., Wasil, E.A. (Eds.), The Vehicle Routing Problem. Latest Advances and New Challenges. Springer, New York, pp. 359–377.

Gutierrez-Jarpa, G., Marianov, V., Obreque, C., (2009). A single vehicle routing problem with fixed distribution and optional collections. IIE Transactions 41, 1067–1079.

Gutierrez-Jarpa, G., Desaulniers, G., Laporte, G., Marianov, V. (2010). A branch-and-price algorithm for the Vehicle Routing Problem with Deliveries, Selective Pickups and Time Windows, European Journal of Operational Research, 206(2), 341–349.

Hernandez-Perez, H., Salazar-Gonzalez, J.-J. (2007). The one-commodity pickup- and-delivery traveling salesman problem: inequalities and algorithms. Networks, 50 (4), 258–272.

Li, Y., Chen, H., Prins, C. (2016) Adaptive large neighborhood search for the pickup and delivery problem with time windows, profits, and reserved requests. European Journal of Operational Research, 252(1), 27–38.

Nalepa, J., Blocho, M. (2016). Enhanced guided ejection search for the pickup and delivery problem with time windows. In: Asian Conference on Intelligent Information and Database Systems, Springer, pp. 388–398.

Nguyen, PK., Crainic, TG., Toulouse, M. (2015). Multi-trip pickup and delivery problem with time windows and synchronization. Annals of Operations Research, 1–36.

Parragh, S.N., Dorner, K.F., Hartl, R.F. (2008). A survey on pickup and delivery models : Part II: transportation between pickup and delivery locations. J. Betrieb- swirtschaft 58, 81–117.

Prive, J., Renaud, J., Boctor, F.F., Laporte, G.(2006). Solving a vehicle routing problem arising in soft-drink distribution. Journal of the Operational Research Society. 57, 1045–1052.

Qiu, X., Feuerriegel, S. (2014). A multi-vehicle profitable pickup and delivery selection problem with time windows. In: Proceedings of the European Conference on Information Systems (ECIS).

Qiu, X., Feuerriegel, S., Neumann, D. (2017). Making the most of fleets: a profit-maximizing multi-vehicle pickup and delivery selection problem. Eur. J. Oper. Res. 259 (1), 155–168.

Riedler, M., Raidl, G. (2018). Solving a selective dial-a-ride problem with logic-based Benders decomposition. Computers and Operations Research. 96, 30–54.

Ropke, S., Pisinger, D. (2006). An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. Transp. Sci., 40 (4), 455-472.

Ropke, S., Cordeau, J.-F. (2009). Branch and cut and price for the pickup and delivery problem with time windows. Transp. Sci. 43 (3), 267–286.

Tas, D., Gendreau, M., Dellaert, N., van Woensel, T., de Kok, A.G. (2014) Vehicle routing with soft time windows and stochastic travel times: A column generation and branch-and-price solution approach. European Journal of Operational Research. 233(3), 789–799.

Ting, C.K., Liao, X.L. (2013) The selective pickup and delivery problem: Formulation and a memetic algorithm, Int. J. Prod. Econ. 141 (1), 199–211.